Quiz 3

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1. (a) Draw the subgroup lattice of $\mathbb{Z}_{30}$.

(b) Make a table with all the elements of $\mathbb{Z}_{30}$, grouped according to their orders; how many elements of each possible order are there?

2. Let a be an element of a group G, and suppose a has order 24.

(a) List all the elements in the subgroup $\langle a^4 \rangle$, together with their respective orders.

(b) What are the generators of the subgroup $\langle a^4 \rangle$?

3. Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 6 & 1 & 4 & 2 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{bmatrix}$, viewed as elements in S_6.

(a) Compute the product of α and β:

$\alpha\beta =$

(b) Compute the inverse of α:

$\alpha^{-1} =$

(c) Compute the conjugate of β by α:

$\alpha\beta\alpha^{-1} =$

(d) Do α and β commute?

4. Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 9 & 4 & 7 & 1 & 2 & 8 & 5 & 10 & 6 \end{bmatrix}$, viewed as an element in S_{10}.

(a) Write α as products of disjoint cycles.

(b) Find the order of α.

(c) Write α as a product of transpositions.

(d) Find the parity of α.

5. (a) How many permutations of order 5 are there in S_5?

(b) How many permutations of order 5 are there in S_6?

6. Find permutations α and β such that:

(a) $|\alpha| = 2$, $|\beta| = 2$, and $|\alpha\beta| = 2$.

(b) $|\alpha| = 2$, $|\beta| = 2$, and $|\alpha\beta| = 3$.

(c) $|\alpha| = 2$, $|\beta| = 4$, and $|\alpha\beta| = 4$.