Prof. Alexandru Suciu

MATH 3175 — Group Theory — Fall 2010

Quiz 2

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1. Let G be the group defined by the following Cayley table.

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	3	4	1	6	7	8	5
3	3	4	1	2	7	8	5	6
4	4	1	2	3	8	5	6	7
5	5	8	7	6	1	4	3	2
6	6	5	8	7	2	1	4	3
7	7	6	5	8	3	2	1	4
8	8	7	6	5	4	3	2	1

(a) For each element $a \in G$, find the order $|a|$.

(b) What is the center of G?

2. Let G be an abelian group with identity e, and let H be the set of all elements $x \in G$ that satisfy the equation $x^3 = e$. Prove that H is a subgroup of G.

3. Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, viewed as a 2×2 matrix with entries in $\mathbb{Z}_5$.

(a) Show that A belongs to $\mathrm{GL}_2(\mathbb{Z}_5)$.

(b) Does A belong to $\mathrm{SL}_2(\mathbb{Z}_5)$? Why, or why not?

(c) Find all the elements in the cyclic subgroup $\langle A \rangle$ generated by A.

(d) Find the order of A in $\mathrm{GL}_2(\mathbb{Z}_5)$.

4. Let G be a group, H a subgroup of G, and a an element of H. Recall $C(a)$ denotes the centralizer of a, whereas $C(H)$ denotes the centralizer of H.

(a) Show that $C(H) \subseteq C(a)$.

(b) Suppose $H = \langle a \rangle$ is the cyclic subgroup generated by a. Show that $C(\langle a \rangle) = C(a)$.

5. Consider the group $G = \mathbb{Z}_{18}$, with group operation addition modulo 18.

(a) For each element $k \in \mathbb{Z}_{18}$, compute the order of k.

(b) Find all the generators of $\mathbb{Z}_{18}$.

(c) Write all the elements of the subgroup $\langle 3 \rangle$.

(d) Find all the generators of $\langle 3 \rangle$.

6. Let $G = \langle a \rangle$ be a group generated by an element a of order $|a| = 28$.

(a) Is $\langle a \rangle = \langle a^{-1} \rangle$? Is a^{-1} a generator of G? Justify your answers.

(b) Find all elements of G which generate G.

(c) Find an element in G that has order 4. Does this element generate G?

(d) Find the order of a^{12}.