Prof. Alexandru Suciu

MATH 3175 | **Group Theory** | **Summer 2, 2022**

Assignment 5

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1. Let G_1 and G_2 be two groups, with identities e_1 and e_2, respectively. Let $G = G_1 \times G_2$ and let $H = \{(g_1, g_2) \in G_1 \times G_2 : g_2 = e_2\}$. Show that

(i) H is a normal subgroup of G.

(ii) $H \cong G_1$.

(iii) $G/H \cong G_2$.

2. Let H be a subgroup of G, and define its *normalizer* as $N(H) \coloneqq \{g \in G : gHg^{-1} = H\}$.

(i) Show that $N(H)$ is a subgroup of G.

(ii) Show that the subgroups of G that are conjugate to H are in one-to-one correspondence with the left cosets of $N(H)$ in G.

3. Let D_n be the dihedral group of order $2n$. Decide whether the following pairs of groups are isomorphic or not. In each case, explain why.

(i) D_8 and $Q_8 \times \mathbb{Z}_2$.

(ii) D_6 and $S_3 \times \mathbb{Z}_2$.

(iii) D_6 and A_4.

(iv) $D_6 \times \mathbb{Z}_2$ and S_4.

4. Let $G = \mathbb{Z}_4 \times \mathbb{Z}_6$. Compute the factor groups $G/\langle(2, 3)\rangle$ and $G/\langle(3, 3)\rangle$. (In each case, write the result in terms of known finite groups, and explain your answer.)

5. For each of the following groups, find all the conjugacy classes and write out the conjugacy class equation:

(i) D_5.

(ii) D_6.

(iii) A_4.

6. The alternating group A_5 has 5 conjugacy classes, of sizes 1, 12, 12, 15, 20. Use this information to prove that A_5 is a simple group.