For each problem be sure to explain the steps in your argument and fully justify your conclusions.

1. For the power series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(3^{2n})\sqrt[5]{n^3}} x^n$,
   (a) (10 pts) Find the radius of convergence.
   (b) (10 pts) Find the exact interval of convergence.

2. For $n \in \mathbb{N}$, let $f_n(x) = (\sin(x))^n$ and let S equal to the set of real numbers, x, for which $f(x) := \lim_{n\to\infty} f_n(x)$ exists.
   (a) (10 pts) Describe the set S and the function $f(x)$ for $x \in S$.
   (b) (10 pts) For elements y not in S, give an argument that shows $\lim f_n(y)$ does not exist.

3. For $n \in \mathbb{N}$, let $f_n : [0, \infty) \to \mathbb{R}$ be the function given by

$$f_n(x) = \begin{cases} 1 & \text{if } n-1 \leq x \leq n, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) (10 pts) Show that the sequence (f_n) converges pointwise on $[0, \infty)$ and determine the function $f = \lim_{n\to\infty} f_n$.
   (b) (15 pts) Does the sequence (f_n) converge uniformly on $[0, \infty)$?

4. Let $f_n(x) = \dfrac{2n - \cos^2(3x)}{5n + \sin(x)}$.
   (a) (10 pts) Show that (f_n) converges uniformly on $\mathbb{R}$. *Hint*: First decide what the limit function is and then show that convergence is uniform.
   (b) (10 pts) Using your result in part (a) and results in the text, determine $\lim_{n\to\infty} \int_a^b f_n(x)\,dx$ for $a < b$. B sure to cite any results you use to justify your answer.

5. (15 pts) For $n \in \mathbb{N}$, let $f_n : [0, 1] \to \mathbb{R}$ be the function given by $f_n(x) = \displaystyle\sum_{k=0}^{n} \frac{x^k}{2^k}$. Show that the sequence (f_n) is uniformly Cauchy on $[0, 1]$.