In your work on the following problems you may use the theorems about limits in section 9 of the text.

1. (10 pts) Find a function $f(\epsilon)$ defined for $\epsilon > 0$ with the property that
$$\left|\frac{5n+6}{2n-3} - \frac{5}{2}\right| < \epsilon \quad \text{for all } n \in \mathbb{N} \text{ with } n > f(\epsilon).$$

2. (10 pts) Find $\lim \sqrt{9n^2 + 2n - 1} - 3n$.

3. (10 pts) Use the $N - \epsilon$ definition of limit to show that the sequence $a_n = \sin\left(\frac{n\pi}{4}\right)$ does not converge.

4. (10 pts) Prove the *Squeeze Theorem* that if $a_n \leq x_n \leq b_n$ for all $n \in \mathbb{N}$ and $\lim a_n = \lim b_n = L$, then the sequence x_n converges to L.

5. Let x_n be given by $x_1 = 17$ and $x_{n+1} = \sqrt{2x_n + 15}$.
   (a) (10 pts) Show that the sequence x_n is decreasing and bounded below.
   (b) (10 pts) Explain whether the sequence x_n converges or not. If the sequence converges, then find the limit.

6. Consider the following definitions:
   - A sequence $\{a_n\}_{n\geq 1}$ is *eventually* in a set $A \subset \mathbb{R}$ if there exists an $N \in \mathbb{N}$ such that $a_n \in A$ for all $n \geq N$.
   - A sequence $\{a_n\}_{n\geq 1}$ is *frequently* in a set $A \subset \mathbb{R}$ if, for every $N \in \mathbb{N}$, there exists an $n \geq N$ such that $a_n \in A$.

   (a) (10 pts) Is the sequence with terms $a_n = (-1)^n$ eventually or frequently in the set $\{1\}$?
   (b) (10 pts) Which definition is stronger? Does frequently imply eventually or does eventually imply frequently?
   (c) (10 pts) Suppose an infinite number of terms of a sequence $\{x_n\}_{n\geq 1}$ are equal to 2. Is the sequence necessarily eventually in the interval $(1.9, 2.1)$? Is it frequently in $(1.9, 2.1)$?
   (d) (10 pts) Suppose $\lim x_n = 2$. Is the sequence $\{x_n\}_{n\geq 1}$ necessarily eventually in the interval $(1.9, 2.1)$? Is it frequently in $(1.9, 2.1)$?