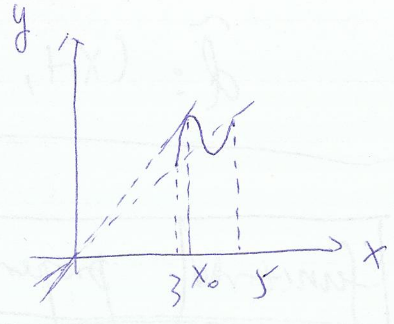
Real Analysis HW #8.

P203. 15) Let $g(x) = \frac{f(x)}{x}$ $g(3) = g(5) = 2$.

$\exists\ x_0 \in (3.5)$ s.t. $g'(x_0) = 0$ ie. $\frac{f'(x_0)x_0 - f(x_0)}{x_0^2} = 0$.

So. the tangent line at x_0. passes through the origin.

P210 (2) $\frac{39}{2}$



(7) $L(f,P) = 0$

Calculate $U(f,P)$

Construct a partition P_n $\forall n \in \mathbb{N}_+$

Take $[0, \frac{1}{n}]$, intervals centered at $\frac{1}{m}$ with length $\frac{1}{m^2 n}$ for $m < n$, and intervals between two adjacents ones of the above.

$U(f, P_n) < \frac{3}{n} \to 0$ as $n \to \infty$.

P211 (8) $|F(x) - F(y)| = \left|\int_x^y f(t)\,dt\right| \leq \left|\int_x^y M\,dt\right| = M|y-x|$

F is a Lipchitz function, Uniformly cont. Of course, it's cont.

P235 (31) $\ln x^{\frac{1}{x-1}} = \frac{\ln x}{x-1}$ $\ln(\lim_{x\to 1} x^{\frac{1}{x-1}}) = \lim_{x\to 1} \frac{\ln x}{x-1} = \lim_{x \to 1} \frac{\frac{1}{x}}{1} = 1$

Hence $\lim_{x\to 1} x^{\frac{1}{x-1}} = e$.

P236 (42) c) $L'(x) = \frac{1}{x}$ by fundamental theorem of caiculus.

a) $L'(x) = \frac{1}{x} > 0$. increasing. (d) $L(1) = \int_1^1 \frac{1}{t}\,dt = 0$

b) Prove that $f(x) = L(xy) - L(x) - L(y) = 0$ for $x > 0$.

$f(1) = L(y) - L(1) - L(y) = 0$

$f'(x) = \frac{1}{xy} \cdot y - \frac{1}{x} - 0 = 0 \quad \forall x > 0$.

f is a contant function so $f(x) = 0 \quad \forall x > 0$.

e) $(L(x) - \ln x)' = 0$.

$(L(x) - \ln(x))\big|_{x=1} = 0$. Hence $L(x) = \ln(x)$.