Real Analysis # 6

P155 2. $A \subset \mathbb{R}^2$ is not closed Hence it's not compact

P155 3. It suffices to prove that it's ~~closed~~ complete and totally bounded.

(i) $cl(A)$ is a closed subset in a complete metric space. Hence, complete

(ii) A totally bounded

$\forall \varepsilon > 0.$ $\exists \{x_i\}_{i=1}^{N} \subset M.$ s.t. $\bigcup_{i=1}^{N} D(x_i, \varepsilon/2) \supset A$

$\forall y \in cl(A)$ $D(y, \varepsilon/2) \cap A \neq \phi.$ Suppose $z \in D(y, \varepsilon/2) \cap A.$

$d(y, z) < \varepsilon/2.$ $d(z, x_i) < \varepsilon/2$ for some i

Hence $d(y, x_i) < \varepsilon.$

So $y \in D(x_i, \varepsilon)$ Hence $\bigcup_{i=1}^{N} D(x_i, \varepsilon) \supset cl(A)$

So $cl(A)$ is totally connected.

P157 4. It is bounded and closed in $\mathbb{R}^n$ Hence compact.

5. $\{\frac{1}{n} \mid n \in \mathbb{N}, n > 0\} \cup \mathbb{Z}$. Not compact

P164. 4. a. Components : $[0, 1]$, $[2, 3]$

b. $\{n\}$ $n \in \mathbb{Z}$

c. $\{x\}$ $x \in \mathbb{Q}$ $0 \le x \le 1$

P173 6. (i) By N.S.P. $\bigcap_{k=1}^{\infty} F_k$ has at least one point.

(ii) Suppose there are two points $x \neq y$. $x, y \in \bigcap_{k=1}^{\infty} F_k$.

Suppose $a = d(x, y) > 0$

diam $F_k \geq a$

$\lim_{k \to \infty}$ diam $F_k \geq a > 0$. Contradiction