Real Analysis Homework #4

P108 4. $C = \bigcup_{y \in B} D_y(1)$

6. $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} < |x_1-x_2| + |y_1-y_2| < \sqrt{2}\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\right)$

$d_S((x_1,y_1),(x_2,y_2)) < d_T((x_1,y_1),(x_2,y_2)) < \sqrt{2}\, d_S((x_1,y_1),(x_2,y_2))$

P109 2. $\mathrm{int}(S) = \{(x,y,z) \in \mathbb{R}^3 \mid 0 < x < 1,\ y^2+z^2 < 1\}$

P112, 2 No $\mathrm{Cl}(S) \neq S$

$\mathrm{Cl}(S) = \{(x,y) \in \mathbb{R}^2 \mid x = 0,\ 0 \le y \le 1\}$

P117 2. $\{0\} \cup \{\frac{1}{n} \mid n = 1, 2, 3, \dots\}$

P120 2. (i) $x \in \mathrm{Cl}(A) \setminus A \Rightarrow x \in \mathrm{Cl}(A)$ & $x \in A^c$

$\Rightarrow x \in \mathrm{Cl}(A)$ & $x \in \mathrm{Cl}(A^c)$

$\Rightarrow x \in \mathrm{Cl}(A) \cap \mathrm{Cl}(A^c) = \mathrm{bd}(A)$

(ii) False.

For example. let $A = [0,1] \subset \mathbb{R}$

$0 \in \mathrm{bd}(A)$ but $0 \notin \phi = \mathrm{Cl}(A) \setminus A$.