Real Analysis #3

P63. 3. $\begin{cases} 3\cdot x + 2y + 2z = 0 \\ 0x + 1\cdot y + 0\cdot z = 0 \end{cases} \Rightarrow \begin{cases} x = -\frac{2}{3}z \\ y = 0 \end{cases}$ The complement is $\left\{ t\begin{pmatrix} -\frac{2}{3} \\ 0 \\ 1 \end{pmatrix} \middle| t \in \mathbb{R} \right\}$

5. Equation of the line

$\begin{cases} x = 1 + t \\ y = 1 + 2t \\ z = 1 + 3t \end{cases}$ $\qquad \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\begin{pmatrix} 1+t \\ 1+2t \\ 1+3t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ has no solution. which means the line doesn't pass 0.

P70. 1. sup norm : $d(f,g) = \sup\{|f(x) - g(x)| \; x \in [0,1]\} = 1$

$\langle f, g\rangle = \int_0^1 f(x)g(x)dx$: $d(f,g) = \|f-g\| = \sqrt{\int_0^1 (f(x)-g(x))^2 dx} = \frac{1}{\sqrt{3}}$.

3. $\langle f, g\rangle = \frac{1}{2}$ $\quad \|f\| = \frac{1}{\sqrt{3}}$ $\quad \|g\| = 1$

4. $(\|f+g\| \leq \|f\| + \|g\|)$

$\|f+g\| = \sqrt{\langle f+g, f+g\rangle} = \sqrt{\int_0^1 (f+g)^2 dx} = \sqrt{\frac{31}{30}}$

$\|f\| = \frac{1}{\sqrt{3}}$. $\|g\| = \frac{1}{\sqrt{5}}$

P98. 12. b. $(\|x+y\| - \|x-y\|)^2 \geq 0 \Rightarrow \|x+y\|^2 + \|x-y\|^2 \geq 2\|x+y\|\|x-y\|$

By parallelogram law $\Rightarrow$ $2\|x\|^2 + \|y\|^2 \geq 2\|x+y\|\|x-y\|$

c. $\|x+y\|^2 - \|x-y\|^2$

$= \langle x+y, x+y\rangle - \langle x-y, x-y\rangle$

$= (\langle x,x\rangle + 2\langle x,y\rangle + \langle y,y\rangle) - (\quad \cdots \quad)$

$= 4\langle x,y\rangle$.