Prof. Alexandru Suciu

MTH G321 **TOPOLOGY 3** **Spring 2009**

HOMEWORK 5

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1. Find a map
$$f\colon K(\mathbb{Z}_2,1) \to K(\mathbb{Z},2)$$
such that
(1) $f_*\colon \widetilde{H}_*(K(\mathbb{Z}_2,1);\mathbb{Z}) \to \widetilde{H}_*(K(\mathbb{Z},2);\mathbb{Z})$ is the zero map, but
(2) $f^*\colon \widetilde{H}_*(K(\mathbb{Z},2);\mathbb{Z}) \to \widetilde{H}_*(K(\mathbb{Z}_2,1);\mathbb{Z})$ is *not* the zero map.
Does this contradict the Universal Coefficient Theorem?

2. Let G be an abelian group, and $K = K(G,n)$, for some $n \geq 1$. Let $\iota \in H^n(K,G)$ be the fundamental class.

(a) Show that there is a map $\mu\colon K \times K \to K$, with $\mu^*(\iota) = \iota \times 1 + 1 \times \iota$.

(b) Show that μ defines an H-space structure on K that is associative and commutative, up to homotopy.

(c) For a CW-complex X, define an addition on $[X,K]$ by $[f]+[g] = [\mu \circ (f \times g)]$. Show that, under the bijection $H^n(X,G) \cong [X,K]$, this addition corresponds to the usual group operation in cohomology.

3. Let $F \to E \xrightarrow{p} B$ be a fibration. Recall a *section* for this fibration is a map $s\colon B \to E$ such that $p \circ s = \mathrm{id}_B$. Recall also that the fibration is *trivial* if there is a fiber-preserving homotopy equivalence $E \simeq F \times B$.

(a) Show that a trivial fibration admits a section.

(b) Show that a *principal* fibration is trivial if and only if it admits a section.

(c) Give an example of a non-trivial fibration which admits a section.

4. Let $X = S^2 \vee S^2$ be the wedge of two 2-spheres. Describe in detail the beginning of the Postnikov tower of X,

$$\begin{array}{ccc} & & X_3 \longleftarrow K(\pi_3(X),3) \\ & \overset{f_3}{\nearrow} & \downarrow \\ X & \xrightarrow{f_2} & X_2 \end{array}$$

In particular, compute $H_5(X_3)$. What does this tell you about the homotopy groups of X?