Prof. Alexandru Suciu

MTH G321 **TOPOLOGY 3** **Spring 2009**

HOMEWORK 4

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1. Let G be an abelian group, and $n > 1$. Show that $H_{n+1}(K(G,n),\mathbb{Z}) = 0$.

2. Let X be a connected CW-complex with $\pi_n(X) = 0$, for all $n \geq 2$. Show that $\pi_n(X^n)$ is a free abelian group, for all $n \geq 2$.

3. Let X be a connected CW-complex, with $\pi_i(X) = 0$ for $1 < i < n$, for some $n \geq 2$. Let $h\colon \pi_n(X) \to H_n(X)$ be the Hurewicz homomorphism. Show that $H_n(X)/h(\pi_n(X)) \cong H_n(K(\pi_1(X), 1)$.

4. Let G be a group, and let $\{M_n\}_{n=1}^{\infty}$ be a sequence of $\mathbb{Z}G$-modules.

(a) Construct a CW-complex X with $\pi_1(X) = G$, and $\pi_n(X) = M_n$ (as $\mathbb{Z}G$-modules).

(b) If $X = K(G,1) \times Y$, where $\pi_1(Y) = 0$, show that $\pi_n(X)$ is trivial as a $\mathbb{Z}G$-module, for all $n > 1$.

5. Let $f = p \circ q\colon T^3 \to S^2$ be the composite of the Hopf map $p\colon S^3 \to S^2$ with the quotient map $q\colon T^3 \to S^3$, collapsing the 2-skeleton of the 3-torus to a point.

(a) Show that $f_* = 0\colon \pi_n(T^3) \to \pi_n(S^3)$, for all $n \geq 1$.

(b) Show that $f_* = 0\colon \widetilde{H}_n(T^3) \to \widetilde{H}_n(S^3)$, for all $n \geq 0$.

(c) Show that, nevertheless, f is *not* homotopic to a constant map.