Finiteness Conditions in Topology and Algebra

Queen's University Belfast, Pure Mathematics Research Centre

31st August - 3rd September

All talks will be in DBB 0G.005

Breaks will be in DBB 0G.007 (the Committee Room)

Time	**Monday**	**Tuesday**	**Wednesday**
9.00 - 9:30	Registration	-	-
9.30 - 10:30	Leary *Uncountably many groups of type FP*	Burghelea *Refinements of basic topological invariants*	Kochloukova *Weak commutativity in groups*
10.30 - 11:00	Break	Break	Break
11.00 - 12:00	Degrijse *Stable finiteness properties of infinite discrete groups*	Suciu *Sigma-invariants and tropical varieties*	Martinez-Pedrosa *A subgroup theorem for homological filling functions*
12.00 - 13:30	Lunch	Lunch	Lunch
13.30 - 14:30	Rahm *Techniques for groups of finite virtual cohomological dimension*	Hüttemann *Homotopy commutative cubes, multicomplexes, and finite domination*	Pettet *Abstract commensurators of the Johnson filtration*
14.30 - 14:45	Break	Break	Break
14.45 - 15:15	Prytuła *Classifying space for virtually cyclic stabilizers for systolic groups*	Steers *Finite domination, Novikov homology and $\mathbb{Z}$-graded rings*	Llosa-Isenrich *A construction method for Kähler groups from maps onto complex tori*
15.15 - 16:00	Break	Break	Break
16.00 - 17:00	Friedl *A marked polytope for 2-generator 1-relator groups*	Geoghegan *The limit set of a G-module controlled over a G-CAT(0) space*	Bridson *Finiteness properties, volume gradients and residually free groups*

Time	**Thursday**
9:30-10:00	Quigley *Regarding the presentation for the commutator subgroup of the Artin group of class C3*
10:00-10:15	Break
10:15-11:15	Shkarin *Finite-dimensional algebras with few relations*
11:15-12:00	Break
12:00-13:00	Witzel *The Basilica Thompson group is not finitely presented*

Abstracts

Martin Bridson Wednesday 16:00	**Finiteness properties, volume gradients and residually free groups** In this talk, I will describe joint work with Kochloukova in which we examine how the number of k-cells required in a minimal $K(G_n, 1)$ grows as one passes to subgroups of increasing finite index in a fixed group G. These volume growth rates bound homology growth rates and are related to L^2 Betti numbers via Lück approximation. We calculate the volume growth of limit groups in all dimensions. For finitely presented residually free groups, we calculate rank gradient, asymptotic deficiency and homology growth rates. Before outlining these calculations, I shall briefly review the theory of limit groups and explain how the finiteness properties of residually free groups are related to their canonical embeddings into direct products of limit groups.
Dan Burghelea Tuesday 9:30	**Refinements of basic topological invariants provided by angle-valued maps (An alternative to Morse-Novikov theory)** The object of attention is the topology of a pair (compact space, degree one integral cohomology class). One describes a class of new computable invariants associated to such pair and to an angle valued map representing the cohomology class of the pair. One discusses their meaning, mathematical properties, and applications/implications in mathematics and outside mathematics.
Dieter Degrijse Monday 11:00	**Stable finiteness properties of infinite discrete groups** Let G be an infinite discrete group. A classifying space for proper actions of G is a proper G-CW-complex X such that the fixed point sets X^H are contractible for all finite subgroups H of G. We consider the stable analogue of the classifying space for proper actions in the category of proper G-spectra and study finiteness properties of such a stable classifying space for proper actions. We investigate when G admits a stable classifying space for proper actions that is finite or of finite type and relate these conditions to classical finiteness properties of the Weyl groups of finite subgroups of G. If G is virtually torsion-free, we show that the smallest possible dimension of a stable classifying space for proper actions coincides with the virtual cohomological dimension of G, thus providing a geometric interpretation of the virtual cohomological dimension of a group.
Stefan Friedl Monday 16:00	**A marked polytope for 2-generator 1-relator groups** We will assign to a marked polytope to a 2-generator 1-relator group. The marked points determine the Bieri-Neumann-Strebel invariant and the shape of the polytope itself contains information about minimal HNN-splittings of the group. Furthermore, if the group is the fundamental group of a 3-manifold, then it determines the Thurston norm of the 3-manifold. This is joint work with Kevin Schreve and Stephan Tillmann.

Ross Geoghegan Tuesday 16:00	**The limit set of a G-module controlled over a G-CAT(0) space** In the late 80's the Bieri-Neumann-Strebel (BNS) invariant of a finitely generated group G appeared. Higher-dimensional analogues, as well as a version for G-modules, were introduced a little later by Bieri-Renz. These invariants have led to deep results in group theory, and they have a fundamental relationship with tropical geometry. Technically, they are subsets of the deleted linear space $V-\{0\}$, where $V := \mathrm{Hom}(G, R)$, but they are better thought of as sets of horospherical limit points associated with the natural action of G on V. Over the last few years Robert Bieri and I have created a substantial generalization of this theory. The linear G-space V is replaced by a proper CAT(0) G-space, and our version leads to interplays with controlled topology, arithmetic groups and their buildings, hyperbolic groups etc. A tantalizing issue is whether these ideas will lead to a non-positively curved version of tropical geometry. I will report and explain.
Thomas Hüttemann Tuesday 13:30	**Homotopy commutative cubes, multicomplexes, and finite domination** A chain complex of R-modules is called R-finitely dominated if it is homotopy equivalent to a bounded complex of finitely generated projective R-modules. (This notion is relevant, for example, in topology of manifolds, group theory, andin the guise of "perfect complexes"in algebraic geometry.) An important special case can be treated explicitly: A bounded chain complex of finitely generated free $R[x, 1/x]$-modules is R-finitely dominated if and only if its "Novikov homology" (homology with coefficients in rings of formal Laurent series) is trivial (Ranicki 1995). In the talk I will discuss a generalisation of this homological criterion for finite domination to cover the case of Laurent polynomial rings in many indeterminates. This is joint work with David Quinn, and ultimately leads to a characterisation of finite domination in terms of "projective toric varieties over non-commutative rings". More precisely I will explain how finite domination implies triviality of Novikov homology. The proof is inspired by a result on totalisation of double chain complexes of Bergman. I will explain how homotopy commutative cubical diagrams occur quite naturally in this context, and how these are related to the formalism of multicomplexes and their totalisations.
Claudio Llosa Isenrich Wednesday 14:45	**A construction method for Kähler groups from maps onto complex tori** In this talk I will describe a method for constructing Kähler groups F. Kähler groups are fundamental groups of compact Kähler manifolds. The idea is to consider a fibration of a compact Kähler manifold X over a complex torus Y. Roughly speaking we prove that if such a fibration has isolated singularities and connected fibers then the fundamental group F of the generic fiber (which is Kähler) fits into a short exact sequence $1 \to F \to G \to A \to 1$, where G is the fundamental group of X and A is the fundamental group of Y. This result was inspired by work of Dimca, Papadima and Suciu: they proved it for Y a torus of complex dimension one and used it to obtain Kähler groups with interesting finiteness properties. It also generalises a Theorem of Shimada.

Dessislava Kochloukova Wednesday 9:30	**Weak commutativity in groups** We revise a construction of a group $H = \chi(G)$ first defined by Sidki in 1980. The group H has a normal abelian subgroup $W(G)$) with quotient a subdirect product of three copies of G. We show some sufficient conditions for $W(G)$ to be finitely generated, hence of homotopical type F_∞. Using results on Sigma theory or recent results on subdirect products due to Bridson, Howie, Miller, Short this unables us to find sufficient conditions for G to be finitely presented. We show further that when G is a soluble group of type FP_∞ then H is a soluble group of type FP_∞. We finish with examples of soluble FP_∞ groups, in one example $W(G)$ is finitely generated and in another is infinitely generated though in all cases H is FP_∞. The work presented is joint work with Saïd Sidki (University of Braslia, Brazil).
Ian Leary Monday 9:30	**Uncountably many groups of type FP** A group G is type F if it admits a finite $K(G,1)$. Since there are only countably many finite group presentations, there are only countably many groups of type F. Roughly speaking, type FP is an 'algebraic shadow' of type F. In the 1990s Bestvina and Brady constructed groups that are type FP but not finitely presented. Since Bestvina-Brady groups occur as subgroups of type F groups, there are only countably many of them. We construct uncountably many groups of type FP. As a corollary, not every group of type FP is a subgroup of a finitely presented group.
Eduardo Martinez Pedroza Wednesday 11:00	**A subgroup theorem for homological filling functions** We use algebraic techniques to study homological filling functions of groups and their subgroups. If G is a group admitting a finite $(n+1)$-dimensional $K(G,1)$ and $H < G$ is of type F_{n+1}, then the n-th homological filling function of H is bounded above by that of G. This contrasts with known examples where such inequality does not hold under weaker conditions on the ambient group G or the subgroup H. We include applications to hyperbolic groups and homotopical filling functions. This is joint work with Gaelan Hanlon, arXiv:1406.1046.
Alexandra Pettet Wednesday 13:30	**Abstract commensurators of the Johnson filtration** The Torelli group is the subgroup of the mapping class group which acts trivially on the homology of the surface. It is the first term of the Johnson filtration, the sequence of subgroups which act trivially on the surface group modulo some term of its lower central series. We prove that the abstract commensurator of each of these subgroups is the full mapping class group. This is joint work with Martin Bridson and Juan Souto.

Tomasz Prytuła Monday 14:45	**Classifying space for virtually cyclic stabilizers for systolic groups** Let G be a group acting properly on a systolic complex X. In this talk I will present the construction of a finite dimensional model for the classifying space $\underline{\underline{E}}G$. Our approach parallels the one used for CAT(0)-groups by W. Lück. The key ingredient is to describe the coarse geometric structure of a *minimal displacement set* of a hyperbolic isometry of X. Namely, we show that this subcomplex of X is quasi-isometric to the product of a tree and a line. This allows us to estimate the dimension of $\underline{\underline{E}}G$ from above by the dimension of X. As a corollary we establish a conjecture of D. Wise: in a systolic group the centralizer of an element of infinite order is commensurable with $F_n \times Z$. This is joint work with D. Osajda.
Ben Quigley Thursday 9:30	**Regarding the presentation for the commutator subgroup of the Artin group of class C3** We give a homological proof that this commutator subgroup is not finitely presented. We use non-crossing partitions to construct a K(G,1) for the group. Then we use a Mayer-Vietorus sequence to calculate the homology of this space and in particular show that it is not finitely generated in dimension 2. Hence via Hopf's integral homology formula the group cannot be finitely presented.
Alexander Rahm Monday 13:30	**Techniques for groups of finite virtual cohomological dimension** The (co)homology of the Bianchi groups has been the subject to a question by Serre, which was open for 40 years, namely on specifying the kernel of the map induced on homology by attaching the Borel-Serre boundary to the symmetric space quotient of the Bianchi groups. This question been given a constructive answer by the speaker. Moreover, the studies of the latter on the (co)homology of the Bianchi groups have given rise to a new technique (called Torsion Subcomplex Reduction) for computing the Farrell-Tate cohomology of discrete groups acting on suitable cell complexes. This technique has not only already yielded general formulae for the cohomology of the tetrahedral Coxeter groups as well as, above the virtual cohomological dimension, of the Bianchi groups (and at odd torsion, more generally of SL_2 groups over arbitrary number fields), it also very recently has allowed Wendt to reach a new perspective on the Quillen conjecture; gaining structural insights and finding a variant that can take account of all known types of counterexamples to the Quillen conjecture. If no counterexample of completely new type surprisingly shows up, then this refined conjecture must be valid.
Stanislav Shkarin Thursday 10:15	**Finite-dimensional algebras with few relations** tba

Luke Steers Tuesday 14:45	**Finite domination, Novikov homology and $\mathbb{Z}$-graded rings** Given rings $R \subseteq K$ and a chain complex C of K-modules, we say C is R-finitely dominated if it is a retract up to homotopy of a bounded, finitely generated R complex. Ranicki and later Hüttemann and Quinn proved a finite domination result for polynomial rings; specifically given a Laurent polynomial ring $R[x, x^{-1}]$, R-finite domination of a chain complex C of $R[x, x^{-1}]$-modules was equivalent to C having trivial Novikov homology. In this talk I will look at the generalisation to strongly $\mathbb{Z}$-graded rings, focusing on showing that trivial Novikov homology implies finite domination. Surprisingly, many of the ideas that provide a proof for polynomial rings can be adapted to work for graded rings. A number of constructions used in the polynomial case, such as quasi-coherent sheaves, are re-defined for the strongly $\mathbb{Z}$-graded case. In particular, this proof will also satisfy twisted polynomial rings as a corollary.
Alex Suciu Tuesday 11:00	**Sigma-invariants and tropical varieties** The Bieri-Neumann-Strebel-Renz invariants $\Sigma^i(X) \subset H^1(X, \mathbf{R})$ of a space X are the vanishing loci for the Novikov homology of X in degrees up to i. In this talk, I will describe a connection between the Σ-invariants of X and the tropicalization of the cohomology support loci $V^i(X) \subset H^1(X, \mathbf{C}^*)$.
Stefan Witzel Thursday 12:00	**The Basilica Thompson group is not finitely presented** Jim Belk and Brad Forrest have constructed a group T_B that acts on the Basilica Julia set much like Thompson's group T acts on the circle. They proved that T_B is finitely generated and virtually simple. I will talk about the joint result with Matt Zaremsky that T_B is not finitely presented. This is an instance of the more general problem of showing that a group G is not of type F_n when the (proper geometric) dimension of G is bigger than n (infinite in this case). In that situation local methods (like combinatorial Morse theory) seem rarely applicable.

Getting around

There are a number of ways to get around Belfast. The 8B and 8C buses goes from the city centre straight to Elm's village and also pass the University along the way. Any 7 bus can take you from the city centre to the university. You can buy your ticket from the bus driver, cost is £1.50 to go from the city centre to the university, and £2.00 to get from the city centre or the university to Elms. You can also get the train from Great Victoria street (beside the Europa hotel) to Botanic and take a left when you leave the station.

Also below is a list of taxi firms that serve all of Belfast city.

Taxis	
Value Cabs	+442890809080
Fonacab	+442890333333
247	+442890247247

Places to eat

There are a number of restaurants on Botanic Avenue, Stranmillis and the Lisburn Road. The students union also has coffee shops and sandwiches available from the SU shop. Cafe Hope is right beside the David Bates building. If you wish to get fresh coffee Common Grounds in the 'Holylands' (leave the David Bates, turn right, left, right) is particularly recommended and if you are a PhD student they give a discount.

Pubs

There are a lot of pubs in Belfast - don't forget it's Ireland! The Students Union speakeasy is very close. On the way back to Elms Village you will past the Botanic and the Eglantine. If you wish to go into the city centre there is a huge choice but the The Crown Liquor Saloon on Great Victoria Street and the Duke Of York in the Cathedral Quarter are excellent.

List of Participants

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