ABSENCE OF GEOMETRIC MODELS IN MEDIEVAL CHINESE ASTRONOMY

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Abstract. In 718 A.D., *Ch'üt'an Hsi-ta* (*Gautama Siddhārtha*), an Indian astronomer who was appointed an "astronomer royal" in the *T'ang* court compiled a compendium of omens and divinations, called *K'aiyüan Chan-ching*, analogous to *Varahamihira*'s *Bṛhatsaṃhita*. The 104th volume of this work, *Chiu-chih li* (Nine Upholders Calendrical System) on astronomy was entirely based on the Indian astronomy of the 7th century which in turn was based on the geometric astronomy of the Greeks. A few years later, the emperor asked *I-hsing* (*Yixing* in *pinyin*), a buddhist monk, an astronomer and a mathematician, to overhaul the traditional Chinese astronomy. He submitted an astronomical system, called *Ta-yen li* (Grand Expansion Calendrical System, *Dayan li* in *pinyin*) in 727 A.D. Only *Ta-yen li* was officially adopted.

In the 13th century, a Persian astronomer submitted to Khubilai Khan an astronomical system, Myriad Year Calendrical System, based on the geometric Islamic astronomy. This too was not officially adopted. Instead, Khubilai Khan ordered a major reform of the traditional Chinese system. The result was *Shou-shih li* (Granting the Seasons Calendrical System) which held sway over the Chinese astronomy until the arrival of the Jesuits in the 17th century.

An analysis of the motion of the Sun and and the lunar parallax in *Ta-yen li* and *Shou-shih li* shows how little impact foreign imports had on Chinese astronomy. Geometric methods began to make inroads in Chinese astronomy only after the arrival of the Jesuits in the 17th century.

1. Introduction

Recorded Chinese mathematical astronomy comes down to us mostly as a part of official dynastic histories. The exceptions are translations of foreign sources such as Buddhist, Islamic and European texts; however these too were handbooks such as Indian *karaṇa* or Islamic *zīj*. These astronomical records, called *lifa* (calendar methods), consist of tables of constants and numerical algorithms for calendric computations. An

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apt translation of *lifa* that is used is "astronomical computistics". A typical chapter in a *lifa* begins by listing the constants used in the chapter. These may be astronomical constants or merely definitions of units used. This is followed by instructions for numerical computation. Here is an example from *Ta-yen li*, VI.15 [Ang1979]:

For the yin phase:
Ecliptic Difference: 1275
Ecliptic Limit: 3524
Possible Ecliptic Limit: 3659
For the yang phase:
Ecliptic Limit: 135
Possible Ecliptic Limit: 974

For the yin phase, subtract while for the yang phase, add the Difference Accumulation for the day in the entry for the day in the entry qui during which the eclipse occurred at conjunction. The result in each case is the true Ecliptic Difference and the true Ecliptic Limit. ...

In this case, Ecliptic Difference is the shift in the lunar node along the ecliptic due to the lunar parallax at the winter solstice as observed in *Yang Ch'eng*. Its value is stated as a known fact. *Ta-yen li* abounds in tables listing equations of the center for the sun and the moon, gnomon shadows, motions of the planets and so on. The whole system is designed not as a treatise on astronomy, but as an instruction manual to be used by the computists in the astronomy bureau.

Since the predictions of celestial phenomena were solely the emperor's prerogative, the *lifa*s were compiled only by royal astronomers. Failed predictions had the potential for undermining the authority of the emperor and hence, frequent revisions were made over time to improve the accuracy. According to Sivin [Sivin2009], there were some 200 systems proposed through history about a quarter of them were officially adopted. Underlying this tradition of repeated revisions was the Chinese belief that nature was inherently irregular, not bound by mathematical laws and frequent revisions were inevitable. Sivin [Sivin1995] quotes *I-hsing* defending mathematical astronomy in the face of this indeterminacy:

"If the anomalies in the celestial positions [of the moon] actually fluctuated with time, providing rebukes [to the ruler] that the regularities of astronomical constants cannot encompass, and substituting for regularity a mutability [that derives] from the inaccessible [fine structure of the

cosmos], this would be a matter beyond even [the ability of] Sages to assess. It can hardly lie within the scope of mathematical astronomy"

More than 5 centuries later, *Shou-shih li* [Sivin2009] has this to say in its introduction:

"Later generations carried on this tradition, through T'ang and Sung, until dozens of experts had appeared who improved the epoch and the techniques. But surely it was not that [they were so frequent merely] because the reformers wanted to differ [from their predecessors]. It would seem, rather, that some irregularities are inherent in the celestial motions, but an astronomical system must use set methods. Thus with the passing of time discrepancies are inevitable. Once they appear, correcting them is unavoidable."

Against this background, Indian astronomy was introduced in China along with Buddhism, beginning in the first century A.D. [Ōhashi2008]. A number of Indian texts were translated into Chinese [Gupta2011]. Initially, the Indian astronomy in China was the Vedic astronomy, but by the time of the *T'ang* dynasty, the classical astronomy of Indian *siddhānta*s was introduced. Of great importance is the compendium, *K'aiyüan Chan-ching*, of *Ch'üt'an Hsi-ta* (*Gautama Siddhārtha*), on omens and divinations, completed in 712 A.D. *Ch'üt'an Hsi-ta* was an Indian astronomer who was appointed an "astronomer royal" in the *T'ang* court. The 104th chapter of *K'aiyüan Chan-ching*, entitled *Chiu-chih li* (Nine-Upholders Calendrical System), is about astronomy [Yabuuti1979]. (*Chiu-chih* is a literal translation of the Sanskrit word *navagraha*, i.e. nine planets.) It is entirely based on the Indian astronomy of the 7th century. In particular, it has a table of sines which is used throughout. It is written purely as a computational manual, with no mention of cosmology, geometric or otherwise. It considers the lunar parallax for the first time in China, but omits discussion of the planets. *Chiu-chih li* was not officially adopted. In fact, *K'aiyüan Chan-ching* was lost until a copy was found around 1600, hidden away in a Buddha statue.

Soon after, the emperor assigned the task of calendar reform to a reluctant *I-hsing* in 721. *I-hsing* was a Buddhist monk, a tantric master. He was well-versed in Sanskrit and translated Sanskrit texts. He was also a skilled mathematician and an astronomer. He designed and constructed a mechanical astronomical clock and an ecliptic-mounted armillary sphere. He carried out detailed astronomical observations. He is famous for his great meridian survey. His culminating achievement was his calendar reform, *Ta-yen li*, (Grand Expansion Calendrical System) completed in 727 A.D. and officially adopted. (See [Ho2000] for a brief biography of *I-hsing*.) The issue is to what extent *I-hsing* and the Chinese astronomy after him were influenced by the Indian astronomy. The *Ta-yen li* refers to the method of solar eclipse prediction transmitted by *Chumolo* (*Kumāra*), a reference to one of the three schools of Indian astronomy in China mentioned by Yabuuti, the other two being *Kāśyapa* and *Gautama*. Yabuuti [Yabuuti1963] also reports mentions of *Kumāra* and *Kāśyapa* in Chinese astronomical texts before *Ta-yen li*. Then there is the plagiarizism controversy. An Indian astronomer, *Ch'üt'an Chuan* lodged a complaint with the emperor that *I-hsing* had plagiarized the *Chiu-chih li*, but the methods were not complete. Now while the Indian system uses the ecliptic coordinate system and divides the celestial perimeter into 360 degrees, *I-hsing* follows the Chinese tradition. He employs the equatorial coordinate system. The distance along the ecliptic is measured in units of *tu* which is the distance travelled by the mean sun in a day. The distance from the ecliptic is measured along the meridians passing through the equatorial pole. The ecliptic pole is entirely absent. Noticing the incompatible coordinate systems, Duan Yao-Yong and Li Wen-Lin [Duan2011] conclude that *Ta-yen li* did not use Indian trigonometry. In this paper, we look at the issue more closely and compare the computations of the solar inequality and the lunar parallax in the two works to conclude that *I-hsing* rejected the Indian system and developed his system entirely within the traditional Chinese paradigm.

Such a rejection was repeated when Islamic astronomy arrived after the Mongol conquests in Central Asia and Iran. A persian astronomer, *Jamāl al-Dīn* built seven astronomical instruments for *Khubilai Khan* in 1267 and submitted to the emperor an astronomical system, the Myriad Year Calendrical System, based on the division of the sky into 360 degrees. In 1273, he was appointed to the post of Acting Director of the Palace Library. Yet, *Jamāl*'s system was not adopted. Instead, *Khubilai Khan* commissioned Chinese astronomers to undertake a major reform of the traditional astronomy. The result was *Shou-shih li* (Granting-the-Seasons Calendrical System) completed in 1280. It was *Shou-shih li* which was promulgated as the official *lifa* and not the Myriad Year system. According to Sivin [Sivin2009], there was a strict social segregation under *Khubilai Khan* and there was not much mixing between the Muslim and Chinese astronomers. *Khubilai* found it politically more expedient to promote the traditional *Shou-shih li* rather than a strange foreign system. After all, Mongols themselves were foreigners trying to establish the legitimacy of their rule. It is known that a large number of scientific books in "western" (meaning Islamic) languages were available in China. Unfortunately, the only information about the activities of the Islamic astronomers available to us is mostly what is gleaned from the records of the *Ming* dynasty which overthrew the *Yüan* dynasty; the most important among these is *Hui-hui li* (Islamic li) [Dalen2002].

Shou-shih li held sway over the Chinese astronomy well into the 17th century when the European astronomy began to make inroads into China. It was the most sophisticated and ambitious reform of Chinese astronomy. We therefore include a comparison of *Ta-yen li* and *Shou-shih li* to trace the consistency of the Chinese tradition.

2. Solar inequality

Let λ and λ' be the mean and true longitude of the sun. Let I denote the equation of the center. Then, ignoring the terms involving higher orders of the eccentricity e of the earth's orbit,

$$I = \lambda' - \lambda = 2e\sin(\lambda - \lambda_0) \tag{2.1}$$

where λ_0 is the longitude of the sun's apogee. Although the traditional Chinese astronomy did not have explicit concept of perigee and apogee, the perigee and apogee of the solar orbit were implicitly assumed to coincide with the winter and summer solstices respectively.

In *Ta-yen li*, a day is divided into 3040 day parts and each day part is divided into 24 fractional parts. A tropical year is assumed to be 1110343 day parts. Therefore, 1 *tu* equals $\frac{360\times3040}{1110343} = 0.9856414°$. The Chinese divide the ecliptic into 24 equal parts called *ch'i*. The mean sun spends 15 days, 664 day parts and 7 fractional parts in each *ch'i*. *I-hsing* tabulates the solar inequality as the number of day parts by which the true sun is ahead of the mean sun at the beginning of each *ch'i*. The solar inequality is assumed to be zero at the winter solstice. The maximum value of the equation of the center in *Ta-yen li* is 7366 day parts which equals 2.388°. Yabuuti [Yabuuti1963] obtains the correct value of the maximum solar inequality in 726 A.D. from S. Newcomb's table as 1.974°.

I-hsing also tabulates the number of day parts by which the true sun gains over the mean sun in each *ch'i*. This is just a table of forward differences ΔI of the table of solar inequality. The graph of ΔI is shown in Figure (1). It shows that ΔI varies almost linearly between the solstices. It should be practically a cosine curve if *I-hsing* had based his calculations on Eq. (2.1). In fact, a cubic interpolates exactly his values of ΔI in each quadrant of the ecliptic. This is not to suggest that *I-hsing* actually formulated such a cubic. In fact, later in the chapter, he uses quadratic interpolation to calculate the values of ΔI at any time in a *ch'i*.

Shou-shih li does explicitly use a cubic to describe daily solar inequality:

$$I = \pm(0.051332x - 0.000246x^2 - 0.00000031x^3) \quad 0 \leq x \leq 88.909225$$

if the true sun is south of the equator and

$$I = \pm(0.048706x - 0.000221x^2 - 0.00000027x^3) \quad 0 \leq x \leq 93.712025$$

if the true sun is north of the equator. x is the distance of the true sun in days from the nearest solstice. The sign is positive if the quadrant contains the vernal equinox, negative otherwise. (*Shou-shih li* does use this degree of precision in its constants by dividing a *tu* into 10^8 parts. It is amazing that the Chinese astronomers insisted on using such a high degree of precision, considering that their formula must have ultimately been based on observations.) The maximum value of the equation of the center is 2.40144161 days or 2.367°. The graph of the corresponding values of ΔI in days/*ch'i* shown in Figure (1) is remarkably close to *I-hsing*'s values. There must have been more accurate contemporary tables of the equation of the center prepared by the Muslim astronomers in the Mongol court. But, as Sivin explains [Sivin2009], "Muslim astronomers came to China because the Mongols wanted second opinions on the reading of the heavenly signs and portents, not because they or their Chinese counterparts wanted scientific exchange."

Nonuniform motion of the sun was first noticed by *Chang Tzu-hsin* in the middle of the 6h century [Yabuuti1963]. *Liu Cho* recorded *Tzu-hsin*'s observations as the number of days spent by the true sun in each *ch'i* in *Huang-chi li* compiled in the late 6th century. These values were repeated in *Lin-te li* in the 7th century. The graph of ΔI derived from this data shown in Figure (1) consists of two segments spanning the equinoxes. Each segment is an oscillating saw-tooth wave function. If we ignore the oscillations, the graph becomes a step-wise constant function. A century later, *I-hsing* has the approximately correct trend line for the equation of the center.

Following the Indian tradition, *Chiu-chih li* assigns 360 degrees to the ecliptic. It sets $\lambda_0 = 80°$ and the maximum solar inequality $2e = 134' = 2.233°$. It lists forward differences of $134 \sin\theta$ at 15 degree intervals for $0 \leq \theta \leq 90°$. We have to convert this data into days spent by the true sun in each *ch'i* in order to compare it with *Ta-yen li*.

$$I = 2.233 \sin(\lambda - 80) \quad \text{degrees}$$

$$\frac{dI}{d\lambda} = \frac{2.233\pi}{180} \cos(\lambda - 80) \;= 0.03898 \cos(\lambda - 80)$$

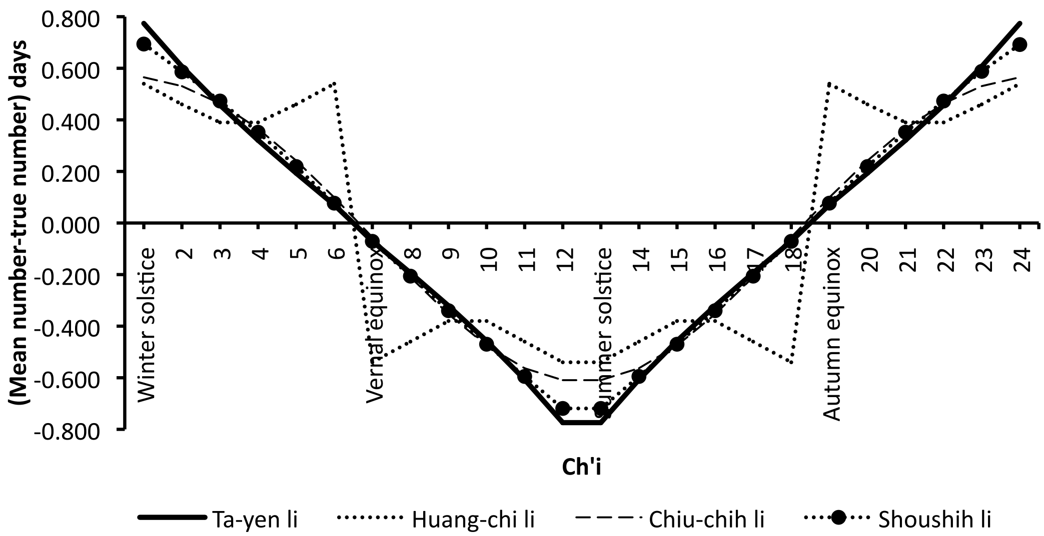

FIGURE 1. Number of days of the mean sun in a ch'i minus the number of days of the true sun in the ch'i

degrees per a degree of travel by the mean sun. Dividing by the velocity $\frac{d\lambda'}{d\lambda}$ of the true sun, we get the rate at which the true sun is gaining over the mean sun in terms of days:

$$\frac{0.03898\cos(\lambda-80)}{1+0.03898\cos(\lambda-80)}$$

showing that the graph of correct ΔI is nearly a cosine curve. The graph of *Chiu-chih*'s values of ΔI as the gain of the true sun over the mean sun in days during a *ch'i* shown in Figure(1) is indeed almost a cosine curve.

3. Solar eclipse and Lunar parallax

Since the lunar orbit is inclined to the ecliptic, the distance between the sun and the moon at the time of conjunction depends on their distance from the lunar nodes. The distance has to be sufficiently small for a solar eclipse to occur. The *observed* distance between the sun and the moon depends on the geographical location of the observer due to the lunar parallax. Therefore, it is essential to take into account the effect of the lunar parallax when predicting solar eclipses.

The lunar parallax p at a location is defined as the angle between the line of sight of the observer looking at the moon and the line joining the moon and the center of the earth. The maximum possible value of p is called the horizontal parallax, H. For all practical purposes, H equals the radius of the earth divided by the distance of the moon from the earth.

$$p \approx \sin p = H \sin z \tag{3.1}$$

where z is the distance of the moon from the zenith. Given the local latitude φ, the declination δ of the moon and the hour angle $\hbar$ of the moon, z may be obtained using the formula

$$\cos z = \sin\varphi \sin\delta + \cos\delta \cos\varphi \cos\hbar$$

from spherical trigonometry.

I-hsing takes into account the lunar parallax in the following manner. The lunar parallax makes the apparent lunar orbit (observed at *Yang Ch'eng*) appear south of the true orbit. Since the lunar orbit is inclined to the ecliptic, the shift in the lunar orbit causes a shift in the lunar node. The distance between the true node and the apparent (observed) node is called the ecliptic difference (*shih-ch'a*). *Ta-yen li* provides values of *shih-ch'a* at the beginning of each *ch'i* in terms of days of moon travel along the ecliptic. The hour angle is not taken into account. The maximum value of the ecliptic difference is given as 1275 day parts and occurs when the sun is farthest from the observer, that is, at the winter solstice. Using the conversion factor given in *Ta-yen li*, we find that the node shifts by $1275 \times \frac{11}{2645} = 5.306$ *tu* $= 5.230°$. *I-hsing* tabulates corrections to be applied to the maximum value of the ecliptic difference to obtain the ecliptic difference at the beginning of each *ch'i*. He also lists forward differences of the corrections which are distributed linearly between the solstices. As a result, the graph of the ecliptic difference consists of two parabolas as shown in Figure (2).

In order to compare *I-hsing*'s values with the theoretical, imagine the moon and the observer to be on the same meridian (i.e. $\hbar = 0$) so that parallax p is confined to the plane of the meridian. It causes a change in the declination of the moon, but has no effect on its right ascension.

$$p \approx \sin p = H \sin(\varphi - \delta) \tag{3.2}$$

The component of p perpendicular to the ecliptic is the parallax in latitude. Let $\alpha =$ the angle between the local meridian and the ecliptic.

Then, from spherical geometry,

$$\tan\alpha = \frac{1}{\cos\ell\tan\epsilon}$$

where ℓ is the longitude of the moon and ϵ is the obliquity of the ecliptic.

$$\text{parallax in latitude} \approx H\sin\alpha\sin(\varphi-\delta) \tag{3.3}$$

and

$$\text{the ecliptic difference} = \frac{\text{parallax in latitude}}{\sin\iota} \tag{3.4}$$

where ι is the inclination of the lunar orbit to the ecliptic. The modern value of the horizontal lunar parallax is about 57'. The latitude of *Yang Ch'eng* is 34.7° and the obliquity of the ecliptic = 23.4°. $\varphi-\delta = 58.1°$. The most sensitive parameter in calculating the ecliptic difference is of course the inclination ι which varies over the years and also depends on the phase of the moon at the time when it passes its nodes. The mean inclination at present is 5.14°. We will set $\iota = 5.15°$, its value in the 13th century [Nakayama1969]. *I-hsing* gives the value of ι as 6 *tu*, or 5.91°. At the winter solstice, the meridian passes through the ecliptic pole and $\alpha = 90$ degrees. The parallax in latitude = 0.8065°. We get the theoretical value of the ecliptic difference equal to 8.961°, about 70% larger than *I-hsing*'s value. The graph of Equation (3.4) at *Yang Ch'eng* is shown in Figure (2). According to Tiezhu Hu [Hu2002], *Ta-yen li* tended to predict too many solar eclipses: It predicted 75% of the actual solar eclipses that occurred in *Xian*, but only about 50% of the predicted eclipses actually occurred.

The parallax formula in *Shou-shih li* takes into account the hour angle, but again it is an empirical formula. As Nakayama [Nakayama1969] remarks, the favorite Chinese practice of approximating an arc by a parabola is very much in evidence. Each formula is designed so that the function smoothly rises to a maximum and then smoothly drops back. Computation begins with the parallax correction for the time of the maximum eclipse:

$$\tau = \frac{\bar{h}(5000-\bar{h})}{9600}$$

where $\bar{h}$ is the time interval between noon and the syzygy, expressed in units of one ten-thousandth of a day. Then, the parallactic shift in the node in *tu* is calculated by combining the following three components:

$$(i)\ \text{North-South difference:}\quad \alpha = \left(4.46 - \frac{\bar{\lambda}^2}{1870}\right)\left(1 - \frac{\bar{h}+\tau}{d/2}\right)$$

where $\bar{\lambda}$ is the longitudinal distance of the true sun from the solstices, and d is the time interval between the sunrise and the sunset, expressed in units of one ten-thousandth of a day.

$$(ii)\ \text{East-West difference:}\quad \beta = \frac{\lambda'(A-\lambda')}{1870}\left(\frac{\bar{h}+\tau}{2500}\right)$$

where A is a semicircle expressed in tu.

$$(iii)\ \text{A constant term}\quad \gamma = 6.15$$

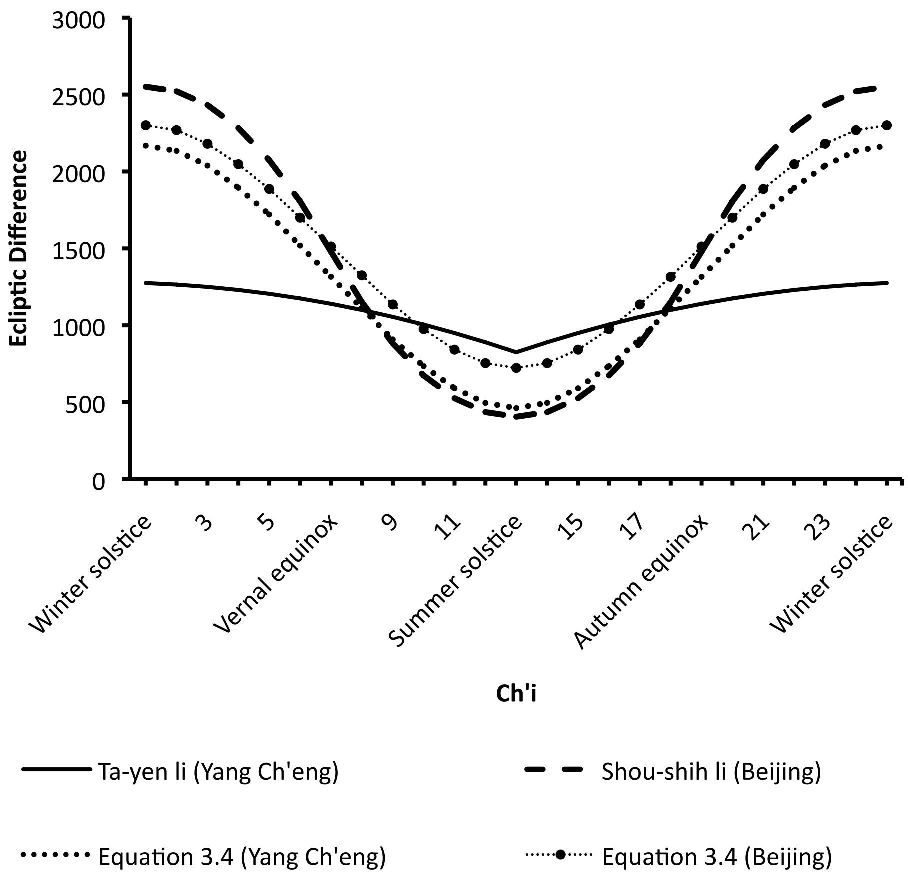

FIGURE 2. Ecliptic Difference: Nodal shift along the ecliptic (hour anlge = 0)

α and β have complementary behavior. α is zero at the equinoxes and has a maximum of 4.46 on solstices at midday. $\beta = 0$ at midday and has a maximum of 4.46 on equinoxes at sunrise and sunset. The first factor of α accounts for the parallax seen an observer on the equator at midday and hence its name. Its second factor gradually reduces the contribution of the "North-South" component as the hour angle increases and the contribution of β increase. The second factor of β is the parallax seen by an observer on the equator on an equinox when the moon is on the equator. Hence it is zero at midday and maximum at sunrise and sunset, giving it the name, "East-West Difference". The contribution of β is gradually reduced to zero as the moon moves towards a solstice.

The constant γ adjusts for local latitude since α and β do not depend on the local latitude. It is the nodal shift as seen from Beijing, the *Yuan* capital, at midday when the moon is on the equator.

The ecliptic difference seen at Beijing at midday according to *Shou-shih li* and its theoretical values according to Equation (3.4) are plotted in Figure (2). The graph consists of two parabolas of opposite concavity, joined smoothly together. Clearly, *Shou-shih li* is a huge improvement over *Ta-yen li*. Li and Zhang [Li1999] have compared the predictions of solar eclipses based on *Shou-shih li* with the modern values to conclude that *Shou-shih*'s accuracy deteriorated with time and it could not compete in accuracy with the *Shih-hsien* calendar devised by the Jesuit astronomers in the 17th century.

Following the Indian *siddhāntas*, *Ch'üt'an Hsi-ta* calculates the lunar parallax using the ecliptic coordinate system, Although *Chiu-chih li* computes the hour angle, it gives a formula for calculating only the parallax in latitude, omitting the parallax in longitude. According to *siddhāntas*,

$$\text{parallax in latitude} = H \sin \zeta \tag{3.5}$$

where ζ is the distance of the ecliptic from the zenith at the time of conjunction. $\zeta = \varphi - \tilde{\delta}$ where $\tilde{\delta}$ is the declination of the highest point on the ecliptic at the time of conjunction. *Chiu-chih li* follows *Khaṇḍakhādyaka* [Sengupta1934] in calculating $\tilde{\delta}$, but uses the erroneous formula $\zeta \approx \varphi - \sin \tilde{\delta}$. (*Pañcasiddhāntikā* [Thibaut1889], *Mahābhāskariya* [Shukla1960] and later, *Sūryasiddhānta* [Burgess2005] use an algorithm different from *Khaṇḍakhādyaka* for calculating $\tilde{\delta}$.)

The rule for the horizontal parallax in *Chiu-chih li* is the one used in all *siddhāntas*: The horizontal parallax of any celestial body is assumed to be one fifteenth of its daily motion. This follows from the assumption that all celestial bodies move with the same linear speed. Then, the angular velocity is inversely proportional to the distance of the celestial body. Its horizontal parallax is also inversely proportional to its distance. Therefore, the horizontal parallax is proportional to the angular velocity. Now *Sūryasiddhānta* states the earth's radius as 800 *yojana*s and the moon's orbit as 324,000 *yojana*s. A linear distance of 800 *yojanas* travelled by the moon corresponds to an angular distance of $800 \times \frac{360}{324000} \times 60 = 53' \; 20"$ = the moon's horizontal parallax. The moon's mean daily sidereal motion is 13° 10' 35" = 790.59'. The time to travel an angular distance of 53' 20" is 53.333/790.59 $\approx \frac{1}{15}$ days. Therefore, the horizontal parallax $\approx$ one fifteenth of the mean daily motion.

At a solstice with $\hbar = 0$, values obtained from Equations (3.3) and (3.5) coincide. At the vernal equinox, we get $0.5117H$ from Equation (3.3) at *Yang Ch'eng*. The formulas in *siddhānta*s give a somewhat different value:

Khaṇḍakhādyaka: $0.4679H$, *Sūryasiddhānta*: $0.4913H$.

4. Concluding Remarks

In this paper, the motion of the sun and the lunar parallax in *Ta-yen li* and *Shou-shih li* are analyzed to assess the influence of imported geometric astronomy on Chinese astronomy during the *T'ang* and *Yuan* dynasties. The analysis shows that there is no evidence of a geometric model in the official Chinese astronomy. The purpose of the official astronomical texts was to provide instructions for calendric computations, relying on observations and interpolation by polynomials. Geometric methods of the Indian buddhists in China are shown to be noticeably more accurate than the traditional algebraic methods, but the Chinese continued to use their algebraic methods with frequent revisions until the arrival of the Jesuits in the 17th century.

I-hsing was a polymath. He must have been aware of *Chiu-chih li*, especially since he is reported to be an expert astrologer and *Chiu-chih li* is a chapter in a larger work, *K'aiyüan Chan-ching* on omens and divinations. Ōhashi [Ōhashi2008] also makes the case that *I-hsing* changed the meaning of days called *mieri* to mean something similar to the omitted *tithi* of the Indian calendar. One can conjecture that it was politically more expedient for the emperor or the Royal Astronomical Bureau to continue to promulgate traditional methods. They must have deemed the degree of accuracy they obtained sufficient for practical and political purposes.

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