Radicals, like fractions, often need to be combined and reduced

EXAMPLES- Combining Radicals with Different with Different Indices

Combining Radicals of the Same Index.

The Goal is to have only a single radical sign. *(assume all variables are positive.)*

Observe that we are Adding Terms with the same Radical....

Examples

a) $\sqrt{50} - \sqrt{8}$
$$= \sqrt{25 \cdot 2} - \sqrt{4 \cdot 2}$$
$$= 5\sqrt{2} - 2\sqrt{2}$$
$$= 3\sqrt{2}$$

b) $\sqrt[3]{24x} - \sqrt[3]{81x}$
$$= \sqrt[3]{8 \cdot 3x} - \sqrt[3]{27 \cdot 3x}$$
$$= 2\sqrt[3]{3x} - 3\sqrt[3]{3x}$$
$$= -\sqrt[3]{3x}$$

c) $\sqrt[4]{4y^3} \cdot \sqrt[4]{12y^2}$
$$= \sqrt[4]{48 \cdot y^5}$$
$$= \sqrt[4]{16y^4 \cdot 3y}$$
$$= 2y\sqrt[4]{3y}$$

Recall: Combining Radicals with Different Indices.

The Goal is to have only a single radical sign. *(assume all variables are positive.)*

Observe that we are Multiplying Terms with the same Exponent....

Examples

a) $\sqrt[3]{2} \cdot \sqrt{3}$

$$
\begin{aligned}
&= 2^{1/3} \cdot 3^{1/2} \\
&= 2^{2/6} \cdot 3^{3/6} \\
&= \left(2^2 \cdot 3^3\right)^{1/6} \\
&= \sqrt[6]{2^2 \cdot 3^3} \\
&= \sqrt[6]{108}
\end{aligned}
$$

b) $\sqrt[3]{y} \cdot \sqrt[4]{2y}$

$$
\begin{aligned}
&= y^{1/3}(2y)^{1/4} \\
&= y^{4/12}(2y)^{3/12} \\
&= \sqrt[12]{y^4 \cdot (2y)^3} \\
&= \sqrt[12]{8y^7}
\end{aligned}
$$

c) $\sqrt{\sqrt[3]{2}}$

$$
\begin{aligned}
&= \left(2^{1/3}\right)^{1/2} \\
&= 2^{1/6} \\
&= \sqrt[6]{2}
\end{aligned}
$$

Getting Expressions to a single base.

Issue. Sometimes we want an exponential expression in terms of a Predefined Base.

Use properties of exponents to write the following expressions in the form 2^{kx} for a suitable k

(a) $4^{5x/2} = (2^2)^{5x/2} = 2^{5x} \quad \Longrightarrow k = 5$

(b) $(2^{4x} \cdot 2^{-x})^{1/2} = (2^{4x-2})^{1/2} = (2^{3x})^{1/2} = 2^{(3/2)x} \quad \Longrightarrow k = \frac{3}{2}$

(c) $\frac{10^x}{5^x} = \frac{(2\cdot5)^x}{5^x} = \frac{2^x \cdot 5^x}{5^x} = 2^x \quad \Longrightarrow k = 1$

(d) $8^{x/3} \cdot 16^{3x/4} = (2^3)^{x/3} \cdot (2^4)^{3x/4} = 2^x \cdot 2^{3x} = 2^{4x} \quad \Longrightarrow k = 4$

(e) *usage:* Solve $7 \cdot 2^{6-3x} = 28$

$\Rightarrow 2^{6-3x} = 4 = 2^2$.

Since the **Bases are the Same** $\Rightarrow 6 - 3x = 2$

Hence $x = 4/3$

Issue. Sometimes we want an exponential expression with just a single x in the exponent.

(a) Determine the base b so that $5^{3x} = b^x$.

$5^{3x} = (5^3)^x = 125^x \Rightarrow b = 125$