Handout Exercises - More Exponent and Radical Expressions

NOTE - there will be material from the previous quiz.

Remark: Recall that fractions should be combined into one fraction and reduced. Often radicals need to be combined into one radical and reduced.

From Previous handout: Rationalize the denominator *Observe - variables*

a) $\dfrac{2}{\sqrt[3]{x}}$ b) $\dfrac{x}{y^{2/5}}$ c) $\dfrac{1}{\sqrt{x}-\sqrt{y}}$

From Previous handout: Reduce and Rewrite each expression using a single radical sign. *Observe - different indices need to be combined*

a) $\sqrt[3]{3}\cdot\sqrt{2}$ b) $\sqrt{5}\cdot\sqrt[3]{4}$

c) $\sqrt[3]{\sqrt{7}}$ d) $\sqrt[3]{\sqrt[3]{2a}}$

Answers: a) $2\sqrt[3]{x^2}/x$; b) $x\sqrt[5]{y^3}/y = xy^{3/5}/y$; c) $(\sqrt{x}+\sqrt{y})/(x-y)$;

Answers: a) $\sqrt[6]{72}$; b) $\sqrt[6]{2000}$; c) $\sqrt[6]{7}$; d) $\sqrt[9]{2a}$;

Prime decomposition of integers

(a) There are twenty-five prime numbers numbers below one hundred. You should be able to find them all. (Note, one is not considered a prime number and neither is zero.)

Find the prime decomposition of the following integers

a) 68

b) 87

c) 69

d) 94

a) 156

b) 4581

c) $11,830$

d) 1175

Answers a) $2^2 \cdot 17$; b) $3 \cdot 29$; c) $3 \cdot 23$; d) $2 \cdot 47$

Answers a) $2^2 \cdot 3 \cdot 13$ b) $9 \cdot 509$; c) $2 \cdot 5 \cdot 7 \cdot 13^2$; d) $5^2 \cdot 47$;

Simplify and Reduce the following fractional expressions (check by calculator)

a) $\dfrac{7+\frac{3}{8}}{\frac{2}{7}+8}$

b) $\dfrac{5}{4}+\dfrac{4}{3+\frac{3}{7}}$

c) $\dfrac{13}{2}+\dfrac{5}{2}-\dfrac{85}{10}$

d) $\dfrac{17}{12}+\dfrac{13}{12}-\dfrac{75}{36}$

Simplify the exponential expression

a) $\dfrac{x^{19/6}}{\sqrt[6]{x^9}}\cdot\left(x^{-252}\right)^{1/84}$

b) $\dfrac{x^{13/6}}{\sqrt[6]{x^4}}\cdot\left(x^{-240}\right)^{1/90}$

Exercise: Factor out the Greatest Common Factor.

a) $48\sqrt{11}-24\sqrt{77}$

b) $48\sqrt{26}-30\sqrt{91}$

c) $6x^3-12x^2$

d) $4a-8ab$

e) $3abc-4bcd+5cde$

f) $12x^2y^3z+18xy^4z^3$

g) $24d^3x^2+12x^3y^4-12x^2\sqrt[5]{z^3}$

h) $13\sqrt[5]{zx^7}+13\sqrt[5]{x^{12}y^{25}}-\dfrac{13a^2\sqrt[5]{x^7}}{\sqrt{\pi}}$

Answers a) 413/464; b) 29/12; c) 1/2; d) 5/12;
Answers a) $x^{-4/3}$; b) $x^{-7/6}$;
Answers a) $24\sqrt{11}\cdot(2-\sqrt{7})$; b) $6\sqrt{13}\cdot(8\sqrt{2}-5\sqrt{7})$; c) $6x^2\cdot(x-2)$; d) $4a\cdot(1-2b)$; e) $c\cdot(3ab-4bd+5de)$;
f) $6xy^3z(2x+3yz^2)$; g) $12x^2\cdot(2d^3+xy^4-\sqrt[5]{z^3})$; h) $13\sqrt[5]{x^7}\cdot\left(\sqrt[5]{z}+xy^5-\frac{a^2}{\sqrt{\pi}}\right)$;

Combining to a Predefined Common Base

Rewrite each expression in the form 2^{kx} or 3^{kx} for a suitable k.

a) 27^x

b) $(\sqrt[3]{2})^x$

c) $(\frac{1}{8})^x$

d) $9^{-x/2}$

e) $8^{4x/3}$

f) $27^{-2x/3}$

g) $7^{-x} \cdot 14^x$;

h) $2^{5x/4} \cdot (\frac{1}{2})^x$

i) $3^{-2x} \cdot 3^{5x/2}$

j) $(2^{-3x} \cdot 2^{-2x})^{2/5}$

k) $(9^{1/2} \cdot 9^4)^{x/9}$

l) $(16^{1/4} \cdot 16^{-3/4})^{3x}$

m) Find a number b such that the function $f(x) = 3^{-2x}$ can be written in the form $f(x) = b^x$.

n) Find b so that the function $8^{-x/3} = b^x$ for all x.

Solve the following equations for x. *(x should be an integer or rational.)*

a) $5^{2x} = 5^2$

b) $(2.5)^{2x+1} = (2.5)^5$

c) $10^{1-x} = 100$

d) $3(2.7)^{5x} = 8.1$

e) $(2^{x+1} \cdot 2^{-3})^2 = 2$

f) $2^{3x} = 4 \cdot 2^{5x}$

Answers a) 3^{3x}; b) $2^{1/3)x}$; c) 2^{-3x}; d) 3^{-x}; e) 2^{4x}; f) 3^{-2x}; g) 2^x; h) $2^{(1/4)x}$; i) $3^{(1/2)x}$; j) 2^{-2x}; k) 3^x; l) 2^{-6x}; m) $b = 1/9$; n) $b = 1/2$;

Answers a) $x = 1$; b) $x = 2$; c) $x = -1$; d) $x = 1/5$; e) $x = 5/2$; f) $x = -1$;

1a. Simplify the exponential expression: $\dfrac{x^{11/6}}{\sqrt[6]{x^8}} \cdot \left(x^{-209}\right)^{1/66}$

1b. Solve for x: $3x^{1/3} + 33 = 42$

2a. Rationalize the denominator: $\sqrt{\dfrac{2x-4}{x+1}}$

2b. Rationalize the denominator so that it is an integer power of x: $\dfrac{19x^{9/5} - 2}{x^{3/5}}$

3. Simplify *(reduce)* the radical expression: $\sqrt[7]{x^2y^4}\sqrt[7]{x^{10}y^3} =$

4. Rewrite the expression in the form 11^{kx}, for a suitable constant k. $\left(11^{3x} \cdot 11^{6x}\right)^{\frac{4}{5}}$

5. Simplify and Rewrite the following as a single radical. $\sqrt[5]{4} \cdot \sqrt[7]{3}$

6a. Factor the greatest common factor $21\sqrt{11} + 15\sqrt{55}$

6b. Factor the greatest common factor $38d^3\sqrt[5]{x} + 19x^{6/5}y^5 - 19\sqrt[5]{x}z^{5/2}$

ANSWERS: 1) $a) \frac{1}{x^{8/3}}$ $b) 3^3 = 27$ 2a) $\frac{\sqrt{2x^2-2x-4}}{x+1}$ 2b) $\frac{19x^{11/5}-2x^{2/5}}{x}$ 3) $xy\sqrt[7]{x^5}$

4) $k = \frac{36}{5}$; $\left(11^{3x} \cdot 11^{6x}\right)^{\frac{4}{5}} = 11^{\frac{36x}{5}}$ 5) $\sqrt[35]{4^7 3^5} = \sqrt[35]{3,981,312}$

6a) $3\sqrt{11} \cdot \left(7 + 5\sqrt{5}\right)$ 6b) $19\sqrt[5]{x} \cdot \left(2d^3 + xy^5 - z^{5/2}\right)$

7a. Simplify the exponential expression: $\dfrac{x^{18/5}}{\sqrt[5]{x^7}} \cdot \left(x^{-190}\right)^{1/50}$

7b. Solve for x: $2x^{1/5} + 28 = 38$

8a. Rationalize the denominator: $\sqrt{\dfrac{3x-4}{x+4}}$

8b. Rationalize the denominator so that it is an integer power of x: $\dfrac{15x^{4/5}-1}{x^{2/5}}$

9. Simplify *(reduce)* the radical expression: $\sqrt[5]{x^3y^3}\sqrt[5]{x^{14}y^4} =$

10. Rewrite the expression in the form 3^{kx}, for a suitable constant k. $\left(3^{4x} \cdot 3^{-4x}\right)^{\frac{3}{5}}$

11. Simplify and Rewrite the following as a single radical. $\sqrt[3]{4} \cdot \sqrt[5]{2}$

12a. Factor the greatest common factor $24\sqrt{13} - 12\sqrt{65}$

12b. Factor the greatest common factor $-\dfrac{30x^{9/2}}{\sqrt[5]{a}} + 15x^{11/2}y + 15x^{9/2}z^{5/3}$

ANSWERS: 7) $a)\frac{1}{x^{8/5}}$ $b)5^5 = 3125$ 8a) $\frac{\sqrt{3x^2+8x-16}}{x+4}$ 8b) $\frac{15x^{7/5}-x^{3/5}}{x}$ 9) $x^3y\sqrt[5]{x^2y^2}$

10) $k = 0$; $\left(3^{4x} \cdot 3^{-4x}\right)^{\frac{3}{5}} = 3^0$ 11) $\sqrt[15]{4^5 2^3} = \sqrt[15]{8,192}$

12a) $12\sqrt{13} \cdot \left(1 - \sqrt{5}\right)$ 12b) $15x^{9/2} \cdot \left(-\frac{2}{\sqrt[5]{a}} + xy + z^{5/3}\right)$