

Factoring Polynomials

Common Products

$$(U + V)^2 = U^2 + 2UV + V^2$$

Squaring a Sum

$$(U - V)^2 = U^2 - 2UV + V^2$$

Squaring a Difference

$$U^2 - V^2 = (U + V)(U - V)$$

Difference of Two Squares

$$(U + V)^3 = U^3 + 3U^2V + 3UV^2 + V^3$$

Cubing a Sum

$$(U - V)^3 = U^3 - 3U^2V + 3UV^2 - V^3$$

Cubing a Difference

More than 2 terms Each term in the first expression is multiplied by the entire second expression.

$$(a + b + c)(x + y) = (a)(x + y) + (b)(x + y) + (c)(x + y)$$

Homeworks - Algebra of Polynomials

Find each sum or difference of polynomial. *Combine like-x terms*

a) $(3x^2 - 4x) + (5x^2 + 7x - 1)$

b) $(4x^2 - 3x) - (9x^2 - 4x + 3)$

c) $(4ax^3 - a^2x) - (5a^2x^3 - 3a^2x + 3)$

d) $(x^2y^2 - 3xy + 2x) - (6x^2y^2 + 4y - 6x)$

Find the product

a) $(a + 9)(a - 2)$

b) $(2y - 1)(3y + 4)$

c) $(2x + 5)^2$

d) $(3x^3 - 2)(5x^3 + 6)$

Find the product

a) $-3a^3(6a^2 - 5a + 2)$

b) $(x - 4)(z + 3)$

c) $(3b^2 - 5b + 2)(b - 3)$

d) $(2x - 1)(4x^2 + 2x + 1)$

e) $(a - b)(a^2 + ab + b^2)$

f) $(a^2 - ab + b^2)(a + b)$

Answers a) $8x^2 + 3x - 1$; b) $-5x^2 + x - 3$; c) $(-5a^2 + 4a)x^3 + 2a^2x - 3$; d) $-5y^2x^2 + (-3y + 8)x - 4y$;

Answers a) $a^2 + 7a - 18$; b) $6y^2 + 5y - 4$; c) $4x^2 + 20x + 25$; d) $15x^6 + 8x^3 - 12$;

Answers a) $-18a^5 + 15a^4 - 6a^3$; b) $xz - 4z + 3x - 12$; c) $3b^3 - 14b^2 + 17b - 6$; d) $8x^3 - 1$; e) $a^3 - b^3$;
f) $a^3 + b^3$;

Multiply the polynomials

a) $(4x^7 + 7x^6 + 6x^2)(12x^5 + 5x)$

b) $(x^{15} + 8x^{14} + 5x^{13})(x^6 + 10x^5)$

c) $(2x^{7/2} + 5x^{5/2} - 7x^{3/2})(7x^4 - 6x^3)$

d) $(8x^{9/2} + 2x^{7/2} + 2x^{11/2})(9x^{15/2} - 6x^{13/2})$

Factoring

Factor the Greatest Common Factor

a) $21x^{10} + 6x^4$

b) $35x^{5/2} + 30x^{11/2}$

Factor by Grouping

c) $12x^3 - 9x^2 + 32x - 24$

d) $21x^3 - 18x^2 + 14x - 12$

Factor

e) $x^2 - 21x + 110$

f) $7x^2 + 8x + 1$

Factor and Simplify

g) $\frac{x^2 + 11x + 24}{x^2 + 5x - 24} \cdot \frac{x^2 - 12x + 27}{x^2 - 7x - 18}$

h) $\frac{x^2 + 7x - 8}{x^2 + 14x + 48} \cdot \frac{x^2 - 2x - 48}{x^2 - 7x - 8}$

Answers a) $48x^{12} + 84x^{11} + 20x^8 + 107x^7 + 30x^3$; b) $x^{21} + 18x^{20} + 85x^{19} + 50x^{18}$; c) $42x^{9/2} + 14x^{15/2} + 23x^{13/2} - 79x^{11/2}$; d) $18x^{13} + 60x^{12} - 30x^{11} - 12x^{10}$;

Answers a) $3x^4(7x^6 + 2)$; b) $5x^{5/2}(6x^3 + 7)$; c) $(4x - 3)(3x^2 + 8)$; d) $(7x - 6)(3x^2 + 2)$; e) $(x - 11)(x - 10)$; f) $(x + 1)(7x + 1)$; g) $\frac{x+3}{x+2}$; h) $\frac{x-1}{x+1}$;

Solve for x by factoring.

a) $x^2 + 6x - 55 = 0$

b) $x^2 + 13x + 26 = 20x + 34$

c) $7x^2 + 15x + 2 = 0$

d) $13x^2 - 4x + 11 = 9x^2 + 7x + 14$

Solve for x .

a) $4\sqrt{x} = x - 12$

b) $3\sqrt{x} = x - 10$

Find the points of intersection of the graphs of the two functions.

a) $y = x^2 - 10, \quad y = 10 - x$

b) $y = x^2 - 6, \quad y = 9 - 2x$

Solve for x by Completing the Square.

a) $x^2 + 6x + 2 = 0$

b) $x^2 - 6x + 3 = 0$

Answers a) $x = \{-11, 5\}$; b) $\{-1, 8\}$; c) $x = \{-\frac{1}{7}, -2\}$; d) $x = \{-\frac{1}{4}, 3\}$;

Answers a) $x = 36$; b) $x = 25$;

Answers a) $(x, y) = \{-5, 15\}, \{4, 6\}$; b) $(x, y) = \{-5, 19\}, \{3, 3\}$;

Answers a) $x = -3 \pm \sqrt{7}$; b) $x = 3 \pm \sqrt{6}$;

Completely Solve for x

a) $x^3 - 5x^2 - 14x = 0$

b) $x^4 + 18x^3 + 77x^2 = 0$

Solve for x

a) $-\frac{5}{6(x+3)} - \frac{85}{6(x-9)} = 1$

b) $-\frac{4}{3(x-2)} - \frac{14}{3(x-5)} = 1$

Simplify and Solve for x

a) $\frac{x^2 + 9x + 18}{x^2 + 11x + 24} \cdot \frac{x^2 + 7x - 8}{x^2 + x - 2} = 13$

b) $\frac{x^2 - 9}{x^2 + 9x + 18} \cdot \frac{x^2 + 2x - 24}{x^2 - 3x - 4} = 17$

Solve the quadratic-like equations for x

a) $x^4 - 65x^2 + 64 = 0$

b) $x^4 - 5x^2 + 4 = 0$

c) $x^4 - 9x^2 + 8 = 0$

d) $x^6 - 3x^3 - 54 = 0$

e) $x^4 - 17x^2 + 70 = 0$

f) $x^6 + 2x^3 - 35 = 0$

Answers a) $x = \{0, -2, 7\}$; b) $x = \{0, 0, -11, -7\}$;

Answers a) $x = \{-8, -1\}$; b) $x = \{-2, 3\}$;

Answers a) $x \rightarrow -\frac{5}{3}$; b) $x \rightarrow -\frac{5}{4}$;

Answers a) $\{\pm 1, \pm 8\}$; b) $\{\pm 2, \pm 1\}$; c) $\{2\sqrt{2}, -2\sqrt{2}, 1, -1\}$; d) $\{3^{2/3}, -\sqrt[3]{6}\}$; e) $\{\sqrt{7}, -\sqrt{7}, \sqrt{10}, -\sqrt{10}\}$;

f) $\{\sqrt[3]{5}, -\sqrt[3]{7}\}$;

Calculate the product and simplify

a) $(1 + \sqrt{2})(3 + \sqrt{2})$

b) $(5 + \sqrt{2})(4 - 3\sqrt{2})$

c) $(3\sqrt{2} + \sqrt{3})(2\sqrt{2} - \sqrt{3})$

d) $(\sqrt{5} + \sqrt{3})^2$

Rewrite each improper fraction as a mixed number in simplest form.

a) $\frac{31}{9}$

b) $\frac{14}{5}$

c) $\frac{9}{7}$

d) $\frac{61}{13}$

Express each fraction in the form: $quotient + \frac{remainder}{divisor}$
The degree of remainder should be less than the degree of divisor

a) $\frac{x^2 - 1}{x - 1}$

b) $\frac{x^2 - 2x + 1}{x - 2}$

c) $\frac{2x^2 - 3x + 1}{x}$

d) $\frac{x^2 - x - 9}{x - 3}$

Answers a) $5 + 4\sqrt{2}$; b) $14 - 11\sqrt{2}$; c) $9 - \sqrt{6}$; d) $8 + 2\sqrt{15}$;

Answers a) $3\frac{4}{9}$; b) $2\frac{4}{5}$; c) $1\frac{2}{7}$; d) $4\frac{9}{13}$;

Answers a) $x + 1$; b) $x + \frac{1}{x-2}$; c) $2x - 3 + \frac{1}{x}$; d) $x + 2 + \frac{-3}{x-3}$;

Reduce the Rational Expressions to the form

$$\textit{quotient} + \frac{\textit{remainder}}{\textit{divisor}}$$

a) $\frac{3x + 17}{x + 7}$

b) $\frac{5 - 8x}{x - 1}$

c) $-\frac{3(2x^2 + 16x - 19)}{x + 9}$

d) $\frac{-2x^2 + 11x + 3}{x - 6}$

Answers a) $3 - \frac{4}{x+7}$; b) $-8 - \frac{3}{x-1}$; c) $-6x + 6 + \frac{3}{x+9}$; d) $-2x - 1 - \frac{3}{x-6}$;

More Practice

1. Multiply and Simplify: $(x^7 + 7x^3 - x^2)(6x^5 + 5x^4)$

2. Multiply and Simplify: $(x^{3/2} + 5\sqrt{x} + \frac{2}{\sqrt{x}})(9x^{9/2} - 14x^{7/2})$

3. Solve for x : $x^2 + 5x + 22 = -12x - 48$

4. Solve for x : $5x^2 + 29x - 6 = 0$

5. Solve for x : $-\frac{1}{2(x+6)} - \frac{5}{2(x+2)} = 1$

6. Solve for x (*simplify first*): $\frac{x^2 - 9}{x^2 + 11x + 24} \cdot \frac{x^2 + 15x + 56}{x^2 + 12x + 35} = 12$

7. Solve for x : $x^5 + 10x^4 - 39x^3 = 0$

8. Find all real solutions of: $x^4 - 14x^2 + 33 = 0$

ANSWERS: 1) $6x^{12} + 5x^{11} + 42x^8 + 29x^7 - 5x^6$ 2) $9x^6 + 31x^5 - 52x^4 - 28x^3$ 3) $\{-10, -7\}$
.
4) $x = \left\{\frac{1}{5}, -6\right\}$ 5) $x = \{-7, -4\}$ 6) $x \rightarrow -\frac{63}{11}$ 7) $x = \{0, 0, 0, -13, 3\}$
.
8) $\{\sqrt{3}, -\sqrt{3}, \sqrt{11}, -\sqrt{11}\}$