Product Rule a,b are constants; f,g are functions; y', f', g' denote derivatives.

Function	Derivative	
$y = a \cdot f \cdot g$	$y' = a \cdot f' \cdot g + a \cdot f \cdot g'$	**Product Rule**
$y = \big(f \cdot g\big)^n$	$y' = n \cdot (f \cdot g)^{n-1} \cdot \big(f' \cdot g + f \cdot g'\big)$	Product-Chain Rule
$y = f^n \cdot g^m$	$y' = n \cdot f' \cdot f^{n-1} \cdot g + m \cdot f \cdot g' \cdot g^{m-1}$	Chain-Product Rule

Ex1a. Use the product rule to find the derivative. $\big(3x + 24\big) \cdot \big(x^8 + 2x^6 - 4x^2\big)$

Answer:

$$\boldsymbol{\big(3\big) \cdot \big(x^8 + 2x^6 - 4x^2\big) + \big(3x + 24\big) \cdot \big(8x^7 + 12x^5 - 8x\big)}$$

$f = 3x + 24 \quad \Rightarrow \quad f' = 3$
$g = x^8 + 2x^6 - 4x^2 \quad \Rightarrow \quad g' = 8x^7 + 12x^5 - 8x$

Ex1b. Use the product rule to find the derivative. $\big(8x^{13/2} + 10x^{7/2}\big) \cdot \big(x^8 + 2x^6 + 3x^2\big)$

Answer:

$$\boldsymbol{\big(52x^{11/2} + 35x^{5/2}\big) \cdot \big(x^8 + 2x^6 + 3x^2\big) + \big(8x^{13/2} + 10x^{7/2}\big) \cdot \big(8x^7 + 12x^5 + 6x\big)}$$

$f = 8x^{13/2} + 10x^{7/2} \quad \Rightarrow \quad f' = 52x^{11/2} + 35x^{5/2}$
$g = x^8 + 2x^6 + 3x^2 \quad \Rightarrow \quad g' = 8x^7 + 12x^5 + 6x$

Ex1c. Use the product rule to find the derivative. $\big(\sin(6x^8 + 12) + 13e^{\frac{2}{x^4} - 14}\big) \cdot \big(18x - 18\big)$

Answer:

$$\boldsymbol{\big(48x^7 \cos(6x^8 + 12) - \frac{104}{x^5} \cdot e^{\frac{2}{x^4} - 14}\big) \cdot \big(18x - 18\big) + \big(\sin(6x^8 + 12) + 13e^{\frac{2}{x^4} - 14}\big) \cdot \big(18\big)}$$

$f = \sin(6x^8 + 12) + 13e^{\frac{2}{x^4} - 14} \quad \Rightarrow \quad f' = 48x^7 \cos(6x^8 + 12) + -\frac{104}{x^5} \cdot e^{\frac{2}{x^4} - 14}$
$g = 18x - 18 \quad \Rightarrow \quad g' = 18$

Exercises

Find the derivatives of the expressions

a) $\big(17x-15\big)\cdot\big(4x^3-29\big)$

b) $\big(14x-14\big)\cdot\big(5x^4-18\big)$

c) $\big(4x^4+x+5\big)\cdot\big(2x^7-4x^4-4x^3\big)$

d) $\big(x^5+x^2-2\big)\cdot\big(5x^6+x^5-2x^2\big)$

e) $\big(15\ln(\frac{3}{x^3}+12)\big)\cdot\big(\cos(6x^6-18)\big)$

f) $\big(2e^{\frac{5}{x}+17}\big)\cdot\big(\cos(3x^5+14)\big)$

g) $\big(\sin(9x^2-16)+6e^{\frac{9}{x^2}-13}\big)\cdot\big(11x+14\big)$

h) $\big(\sin(7x+18)+15e^{\frac{4}{x}+15}\big)\cdot\big(9x+26\big)$

i) $\big(6x^{5/2}-4\sqrt{x}\big)\cdot\big(5x^2+17\big)$

j) $\big(2x^{13/2}+2x^{7/2}\big)\cdot\big(17x^4-24\big)$

Answers a) $\big(17\big)\cdot\big(4x^3-29\big)+\big(17x-15\big)\cdot\big(12x^2\big)$; b) $\big(14\big)\cdot\big(5x^4-18\big)+\big(14x-14\big)\cdot\big(20x^3\big)$;

c) $\big(16x^3+1\big)\cdot\big(2x^7-4x^4-4x^3\big)+\big(4x^4+x+5\big)\cdot\big(14x^6-16x^3-12x^2\big)$;

d) $\big(5x^4+2x\big)\cdot\big(5x^6+x^5-2x^2\big)+\big(x^5+x^2-2\big)\cdot\big(30x^5+5x^4-4x\big)$;

e) $\big(-\frac{135}{x^4}\cdot\frac{1}{\frac{3}{x^3}+12}\big)\cdot\big(\cos(6x^6-18)\big)+\big(15\ln(\frac{3}{x^3}+12)\big)\cdot\big(-36x^5\sin(6x^6-18)\big)$;

f) $\big(-\frac{10}{x^2}\cdot e^{\frac{5}{x}+17}\big)\cdot\big(\cos(3x^5+14)\big)+\big(2e^{\frac{5}{x}+17}\big)\cdot\big(-15x^4\sin(3x^5+14)\big)$;

g) $\big(18x\cos(9x^2-16)-\frac{108}{x^3}\cdot e^{\frac{9}{x^2}-13}\big)\cdot\big(11x+14\big)+\big(\sin(9x^2-16)+6e^{\frac{9}{x^2}-13}\big)\cdot\big(11\big)$;

h) $\big(7\cos(7x+18)-\frac{60}{x^2}\cdot e^{\frac{4}{x}+15}\big)\cdot\big(9x+26\big)+\big(\sin(7x+18)+15e^{\frac{4}{x}+15}\big)\cdot\big(9\big)$;

i) $\big(15x^{3/2}-\frac{2}{\sqrt{x}}\big)\cdot\big(5x^2+17\big)+\big(6x^{5/2}-4\sqrt{x}\big)\cdot\big(10x\big)$;

j) $\big(13x^{11/2}+7x^{5/2}\big)\cdot\big(17x^4-24\big)+\big(2x^{13/2}+2x^{7/2}\big)\cdot\big(68x^3\big)$;

Quotient Rule a,b are constants; f,g are functions; y', f', g' denote derivatives.

Function	Derivative	
$y = \dfrac{f}{g}$	$y' = \dfrac{f' \cdot g - \cdot f \cdot g'}{g^2}$	**Quotient Rule**
$y = \left(\dfrac{f}{g}\right)^n$	$y' = n \cdot \left(\dfrac{f}{g}\right)^{n-1} \cdot \dfrac{f' \cdot g - \cdot f \cdot g'}{g^2}$	Quotient-Chain Rule

Ex2a. Use the quotient rule to find the derivative. $\dfrac{7x-28}{4x^6+x^3-4}$

Answer: $\dfrac{\boldsymbol{(7) \cdot (4x^6+x^3-4) - (7x-28) \cdot (24x^5+3x^2)}}{\boldsymbol{(4x^6+x^3-4)^2}}$

$f = 7x - 28 \quad \Rightarrow \quad f' = 7$
$g = 4x^6 + x^3 - 4 \quad \Rightarrow \quad g' = 24x^5 + 3x^2$

Ex2b. Use the quotient rule to find the derivative. $\dfrac{19x^4+19}{10x^{3/2}-4\sqrt{x}}$

Answer: $\dfrac{\boldsymbol{(76x^3) \cdot (10x^{3/2}-4\sqrt{x}) - (19x^4+19) \cdot (15\sqrt{x} - \frac{2}{\sqrt{x}})}}{\boldsymbol{(10x^{3/2}-4\sqrt{x})^2}}$

$f = 19x^4 + 19 \quad \Rightarrow \quad f' = 76x^3$
$g = 10x^{3/2} - 4\sqrt{x} \quad \Rightarrow \quad g' = 15\sqrt{x} - \frac{2}{\sqrt{x}}$

Ex2c. Use the quotient rule to find the derivative. $\dfrac{\sin(3x+13)}{13e^{5x^3-13}}$

Answer: $\dfrac{\boldsymbol{(3\cos(3x+13)) \cdot (13e^{5x^3-13}) - (\sin(3x+13)) \cdot (195x^2 \cdot e^{5x^3-13})}}{\boldsymbol{(13e^{5x^3-13})^2}}$

$f = \sin(3x+13) \quad \Rightarrow \quad f' = 3\cos(3x+13)$
$g = 13e^{5x^3-13} \quad \Rightarrow \quad g' = 195x^2 \cdot e^{5x^3-13}$

Exercises

Find the derivatives of the expressions

a) $\dfrac{14x+25}{x^5+4x^3+x^2}$

b) $\dfrac{7x-26}{3x^6-5x^4+2x}$

c) $\dfrac{11x^4-29}{8x^{5/2}+2\sqrt{x}}$

d) $\dfrac{16x^2+16}{6x^{7/2}+4x^{5/2}}$

e) $\dfrac{\cos(3x^6-13)}{14e^{\frac{7}{x^4}+11}}$

f) $\dfrac{\sin(9x^7-18)}{11e^{\frac{9}{x^2}-17}}$

Answers a) $\frac{\left(14\right)\cdot\left(x^5+4x^3+x^2\right)-\left(14x+25\right)\cdot\left(5x^4+12x^2+2x\right)}{\left(x^5+4x^3+x^2\right)^2}$;

b) $\frac{\left(7\right)\cdot\left(3x^6-5x^4+2x\right)-\left(7x-26\right)\cdot\left(18x^5-20x^3+2\right)}{\left(3x^6-5x^4+2x\right)^2}$;

c) $\frac{\left(44x^3\right)\cdot\left(8x^{5/2}+2\sqrt{x}\right)-\left(11x^4-29\right)\cdot\left(20x^{3/2}+\frac{1}{\sqrt{x}}\right)}{\left(8x^{5/2}+2\sqrt{x}\right)^2}$;

d) $\frac{\left(32x\right)\cdot\left(6x^{7/2}+4x^{5/2}\right)-\left(16x^2+16\right)\cdot\left(21x^{5/2}+10x^{3/2}\right)}{\left(6x^{7/2}+4x^{5/2}\right)^2}$;

e) $\frac{\left(-18x^5\sin(3x^6-13)\right)\cdot\left(14e^{\frac{7}{x^4}+11}\right)-\left(\cos(3x^6-13)\right)\cdot\left(-\frac{392}{x^5}\cdot e^{\frac{7}{x^4}+11}\right)}{\left(14e^{\frac{7}{x^4}+11}\right)^2}$;

f) $\frac{\left(63x^6\cos(9x^7-18)\right)\cdot\left(11e^{\frac{9}{x^2}-17}\right)-\left(\sin(9x^7-18)\right)\cdot\left(-\frac{198}{x^3}\cdot e^{\frac{9}{x^2}-17}\right)}{\left(11e^{\frac{9}{x^2}-17}\right)^2}$;