Factoring Polynomials

Common Products

$(U+V)^2 = U^2 + 2UV + V^2$	Squaring a Sum
$(U-V)^2 = U^2 - 2UV + V^2$	Squaring a Difference
$U^2 - V^2 = (U+V)(U-V)$	Difference of Two Squares
$(U+V)^3 = U^3 + 3U^2V + 3UV^2 + V^3$	Cubing a Sum
$(U-V)^3 = U^3 - 3U^2V + 3UV^2 - V^3$	Cubing a Difference

Multiply the Polynomials

Example: $(7x+5+4y)(7x+5-4y)$

$$= 49x^2 + 35x - 28xy + 35x + 25 - 20y + 28xy + 20y - 16y^2$$

$$= 49x^2 + 70x - 16y^2 + 25$$

Factor out the Greatest Common Factor

Example: $18x^3 + 27x^2 = 9x^2(2x) + 9x^2(3) = 9x^2(2x+3)$

Example: $x^2(x+3) + 5(x+3) = (x^2+5)(x+3)$

Factor by Grouping Group terms with a common factor

Example: $x^3 + 4x^2 + 3x + 12 = x^2(x+4) + 3(x+4) = (x^2+3)(x+4)$

Example: $x^3 + 5x^2 - 2x - 10 = x^2(x+5) + (-2)(x+5) = (x^2-2)(x+5)$

Factor the Trinomials $x^2 + bx + c$

Find two numbers that multiply to c and add to b

Rewrite bx as a sum - And Group

Example: $x^2 + 6x + 8$

$2 \cdot 4 = 8$ and $2 + 4 = 6 \quad \Rightarrow \quad (x + 2)(x + 4)$

$x^2 + 6x + 8 = x^2 + 2x + 4x + 8 = x(x + 2) + 4(x + 2)$

Example: $x^2 + 3x - 18$

$(-3) \cdot (6) = -18$ and $(-3) + (6) = 3 \quad \Rightarrow \quad (x - 3)(x + 6)$

By GROUPING

$x^2 + 3x - 18 = x^2 - 3x + 6x - 18 = x(x - 3) + 6(x - 3) = (x + 6)(x - 3)$

Example: $x^2 - 5x - 14$

$(2) \cdot (-7) = -14$ and $(2) + (-7) = -5 \quad \Rightarrow \quad (x + 2)(x - 7)$

$x^2 - 5x - 14 = x^2 + 2x - 7x - 14 = x(x + 2) + (-7)(x + 2)$

Example: $x^2 - 13x + 40$

$(-5) \cdot (-8) = 40$ and $(-5) + (-8) = -13 \quad \Rightarrow \quad (x - 5)(x - 8)$

$x^2 - 13x + 40 = x^2 - 5x - 8x + 40 = x(x - 5) + (-8)(x - 5)$

Factor the Trinomials $ax^2 + bx + c$ $(a \neq 1)$

Find two numbers that multiply to $a \cdot c$ *and add to* b

Rewrite bx *as a sum - And Group*

Example: $8x^2 - 10x - 3$

$$a \cdot c = -24 \Rightarrow (-12) \cdot (2) = -24 \text{ and } (-12) + (2) = -10$$

$$\begin{aligned} 8x^2 - 10x - 3 &= 8x^2 - 12x + 2x - 3 \\ &= 4x(2x - 3) + (1)(2x - 3) \\ &= (4x + 1)(2x - 3) \end{aligned}$$

$$\begin{aligned} 8x^2 - 10x - 3 &= 8x^2 + 2x - 12x - 3 \\ &= 2x(4x + 1) + (-3)(4x + 1) \\ &= (2x - 3)(4x + 1) \end{aligned}$$

Factor the Trinomials $ax^2 + bx + c$ $(a \neq 1)$

Find two numbers that multiply to $a \cdot c$ *and add to* b

Rewrite bx *as a sum - And Group*

Example: $6x^2 + 19x - 7$

$$a \cdot c = -42 \Rightarrow (21) \cdot (-2) = -42 \text{ and } (21) + (-2) = 19$$

$$\begin{aligned} 6x^2 + 19x - 7 &= 6x^2 + 21x - 2x - 7 \\ &= 3x(2x + 7) + (-1)(2x + 7) \\ &= (3x - 1)(2x + 7) \end{aligned}$$