Continuous Representations: What goes right and what goes wrong?

Supplementary Slides

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Brandeis University

Joint with Kiyoshi Igusa and Gordana Todorov

Representation Theory and Related Topics Seminar
3 May 2019

Topics

1. Continuous Clusters
2. Continuous Mutation

Topics

1. Continuous Clusters
2. Continuous Mutation

Examples

Example

Let $a < b \in \mathbb{R}$ and

$$\begin{aligned} T = &\{(-\infty, +\infty), (-\infty, a), (a, b), (-\infty, b), (b, +\infty)\} \\ &\cup \{[x, a), \{x\} : -\infty < x < a\} \\ &\cup \{(a, x], \{x\} : a < x < b\} \cup \{(b, x], \{x\} : b < x\}. \end{aligned}$$

T is a continuous cluster with three *continuous fountains*.

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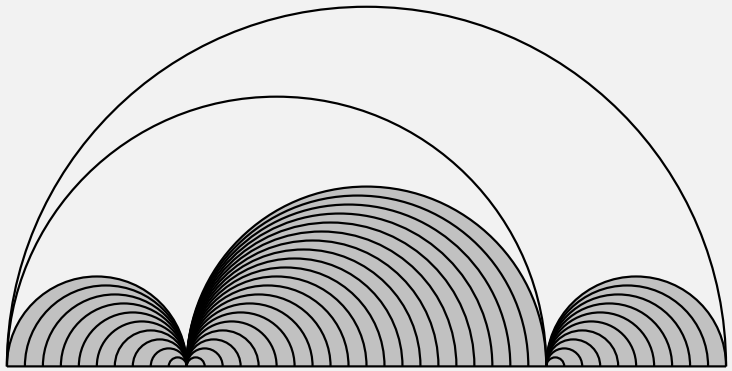
T is a continuous cluster with three *continuous fountains*.
We can visualize T as a set of noncrossing partitions of $\mathbb{R}$:



Structure

The types of compatible sets that make up a continuous cluster are:

- Discrete
- Nests
- Continuous fountains (a type of nest)
- Antifountains (a type of nest)
- Simples

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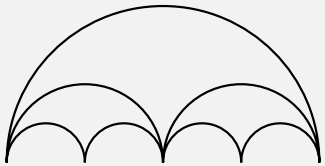
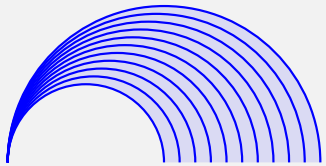
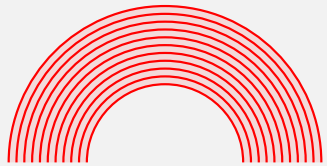


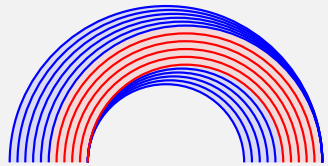
Discrete



Continuous Fountain



Antifountain

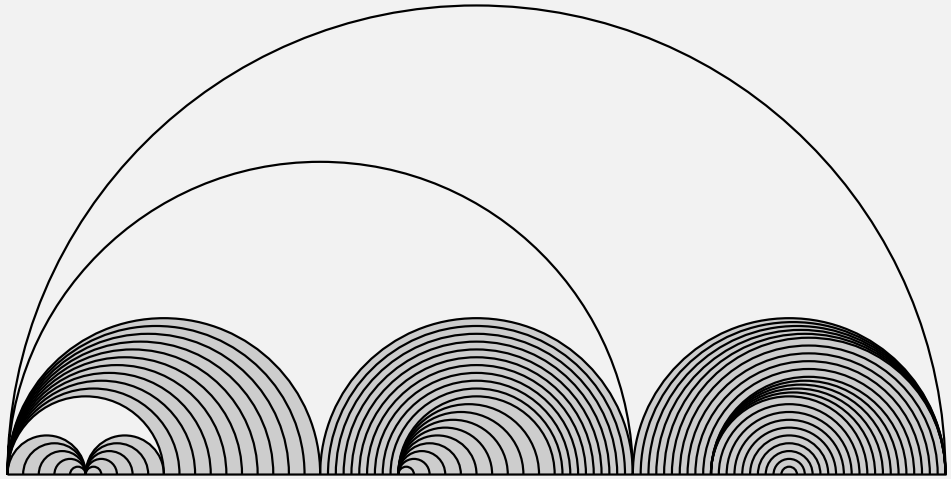


Nest

Structure

Example

Here is another picture of a continuous cluster.



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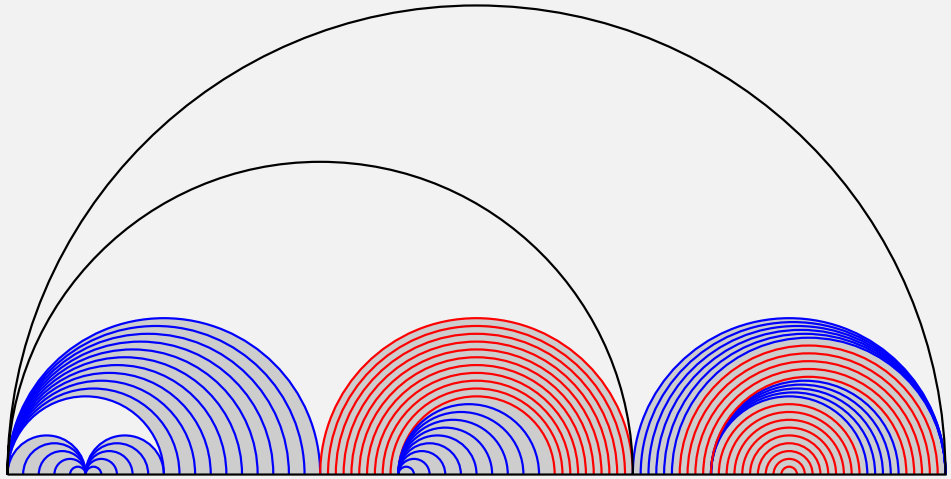

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Example (Continuous Flip)

Let $a, b \in \mathbb{R}$ such that $a < 0 < b$. Let T be

$$\{(-\infty, +\infty), (-\infty, a), (a, b), (-\infty, b), (b, +\infty)\}$$
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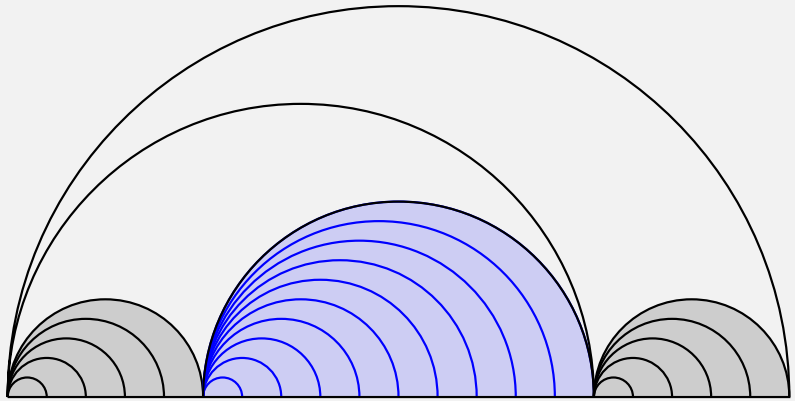
for all $x < a < y < b < z$. We're going to "flip" this blue continuous fountain at a to a purple continuous fountain at b.



$$\mu(a, y] \mapsto [y, b) \text{ and } f(a, y] = \frac{b-y}{b-a} = g[y, b)$$

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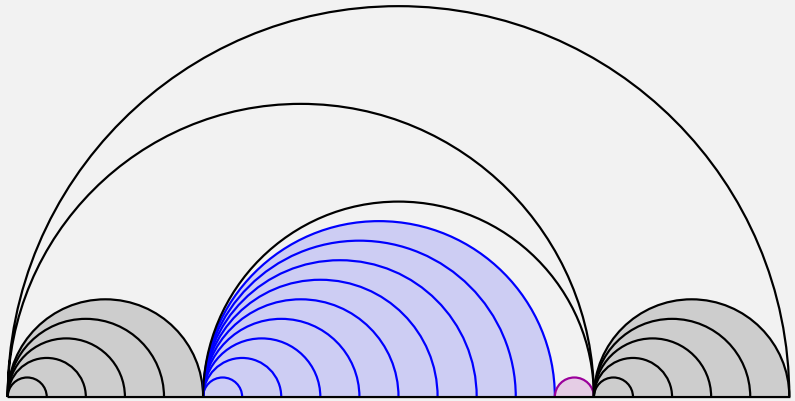

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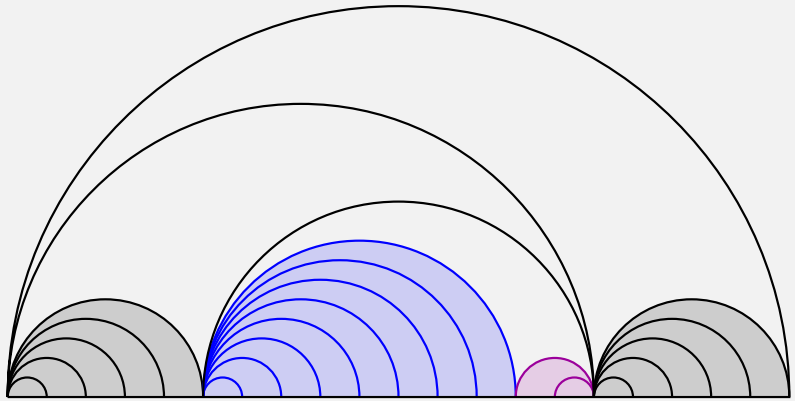

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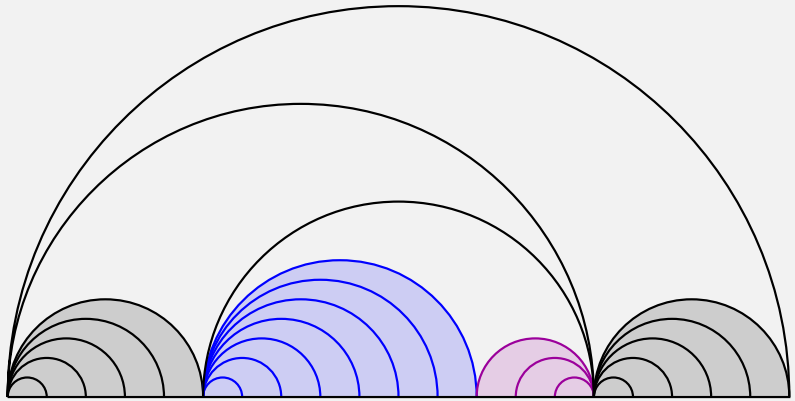

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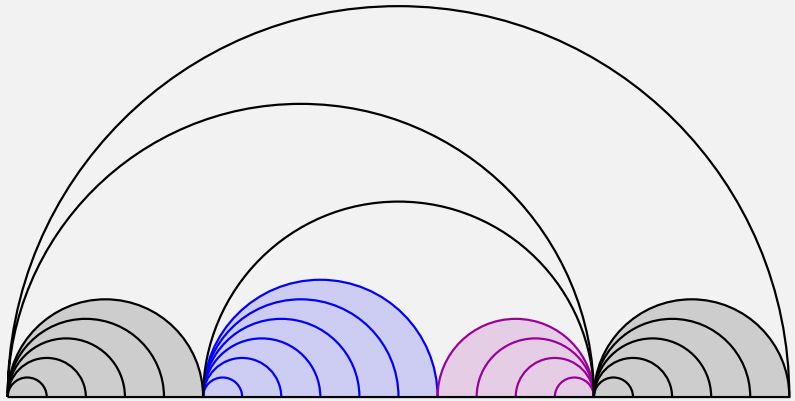

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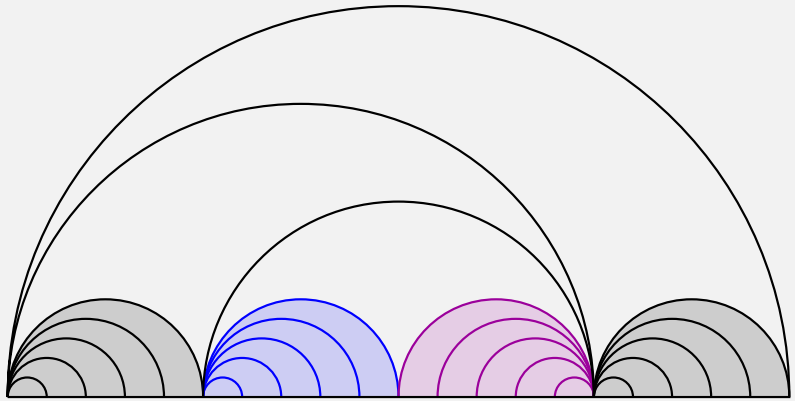

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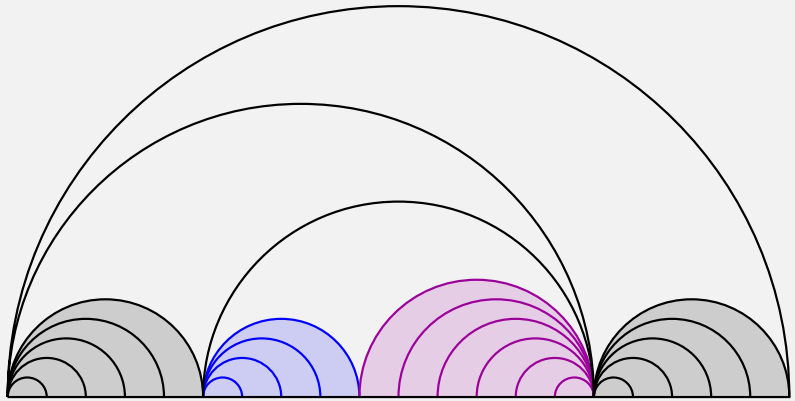

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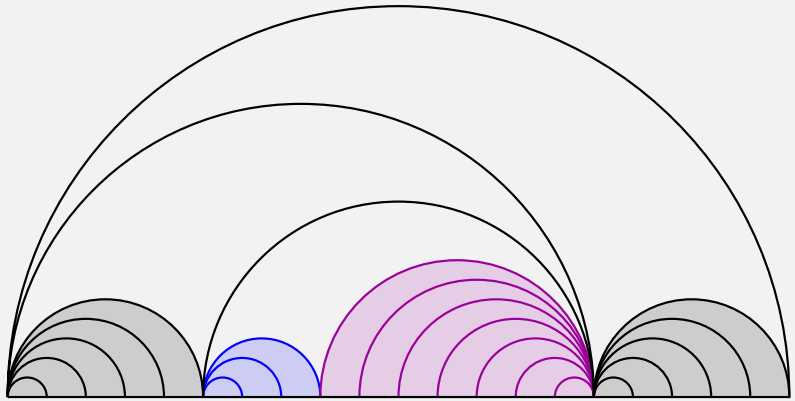

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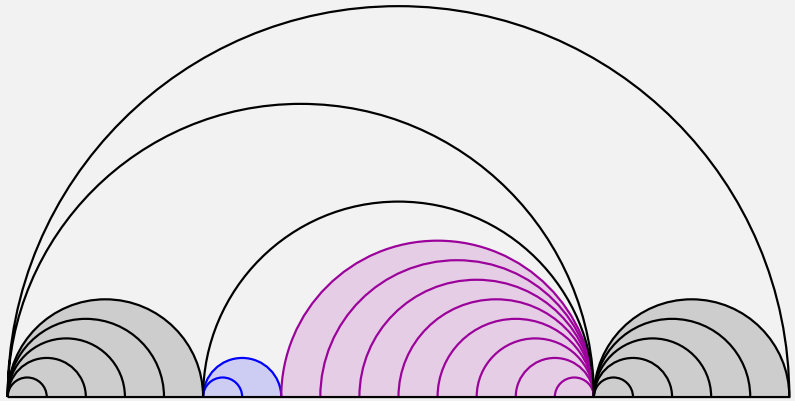

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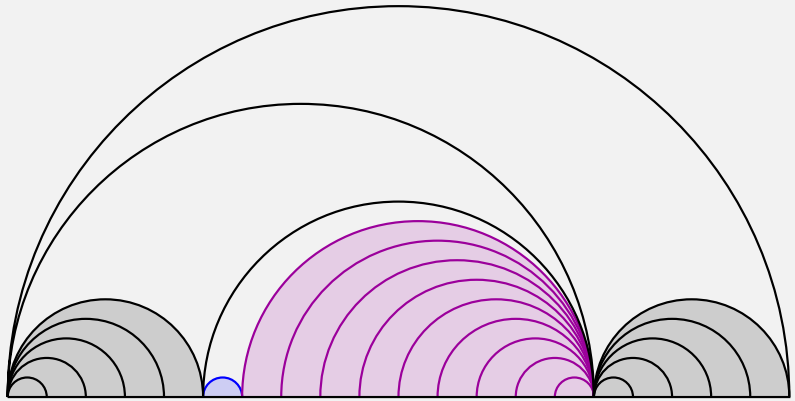

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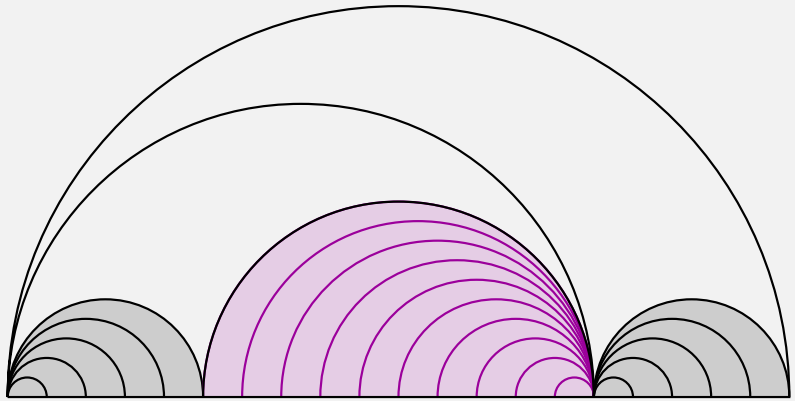

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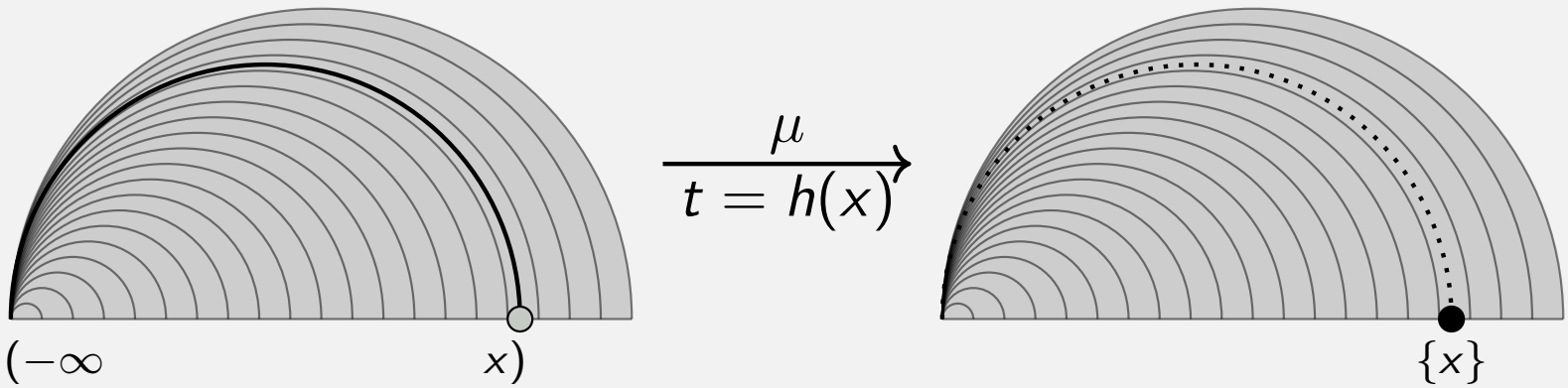
Examples

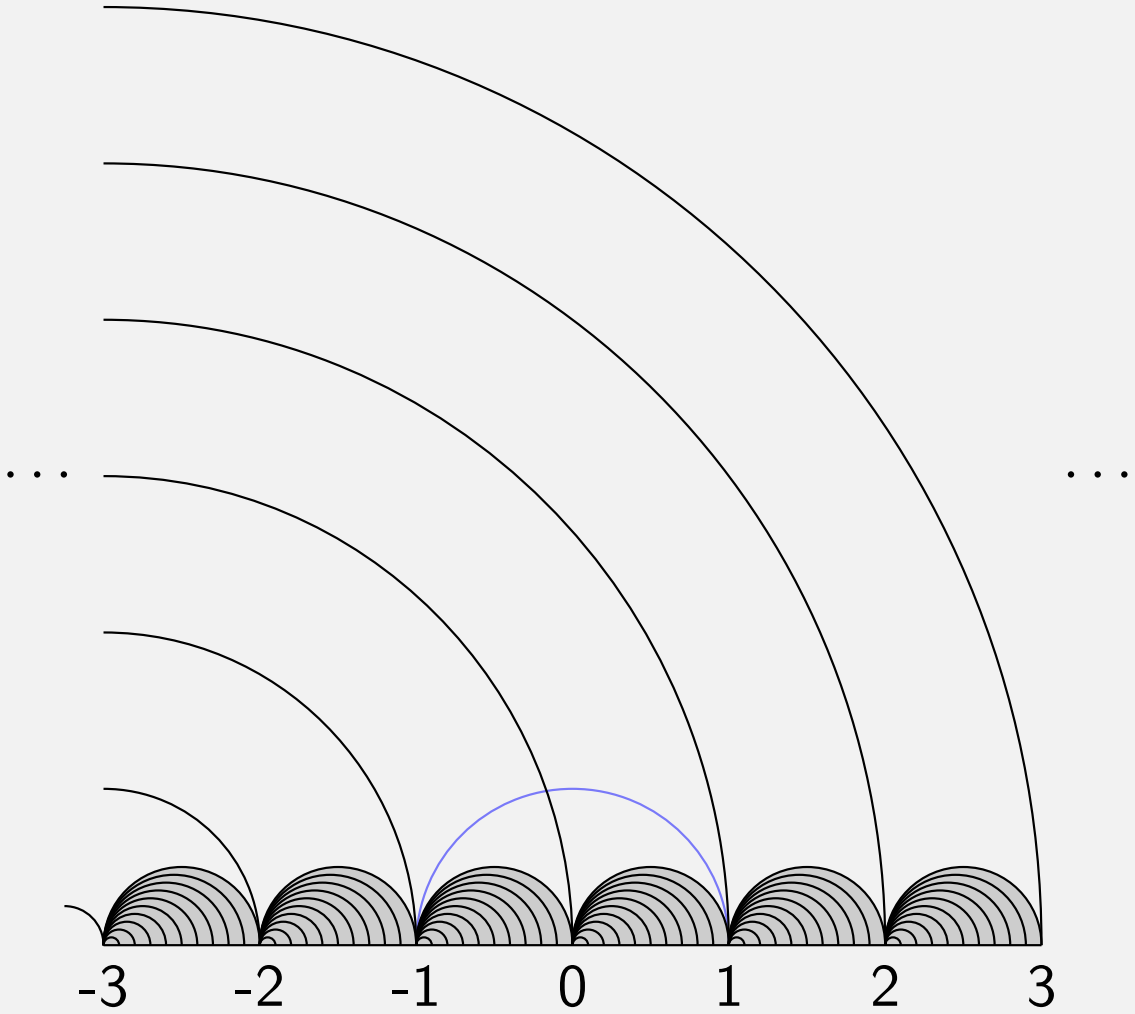
Example (Lamination)

Here is a picture:



Examples

Example (Transfinite Mutation)



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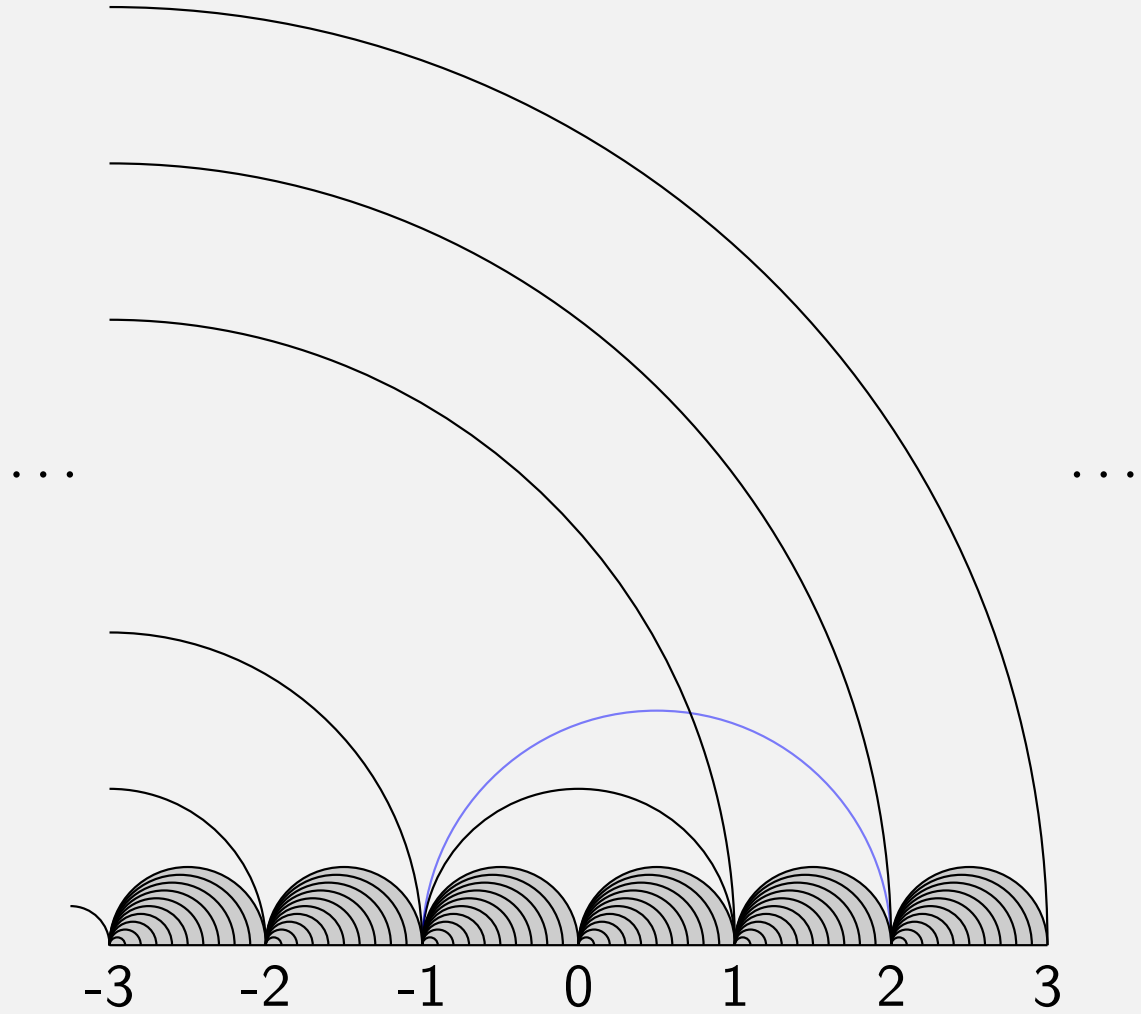

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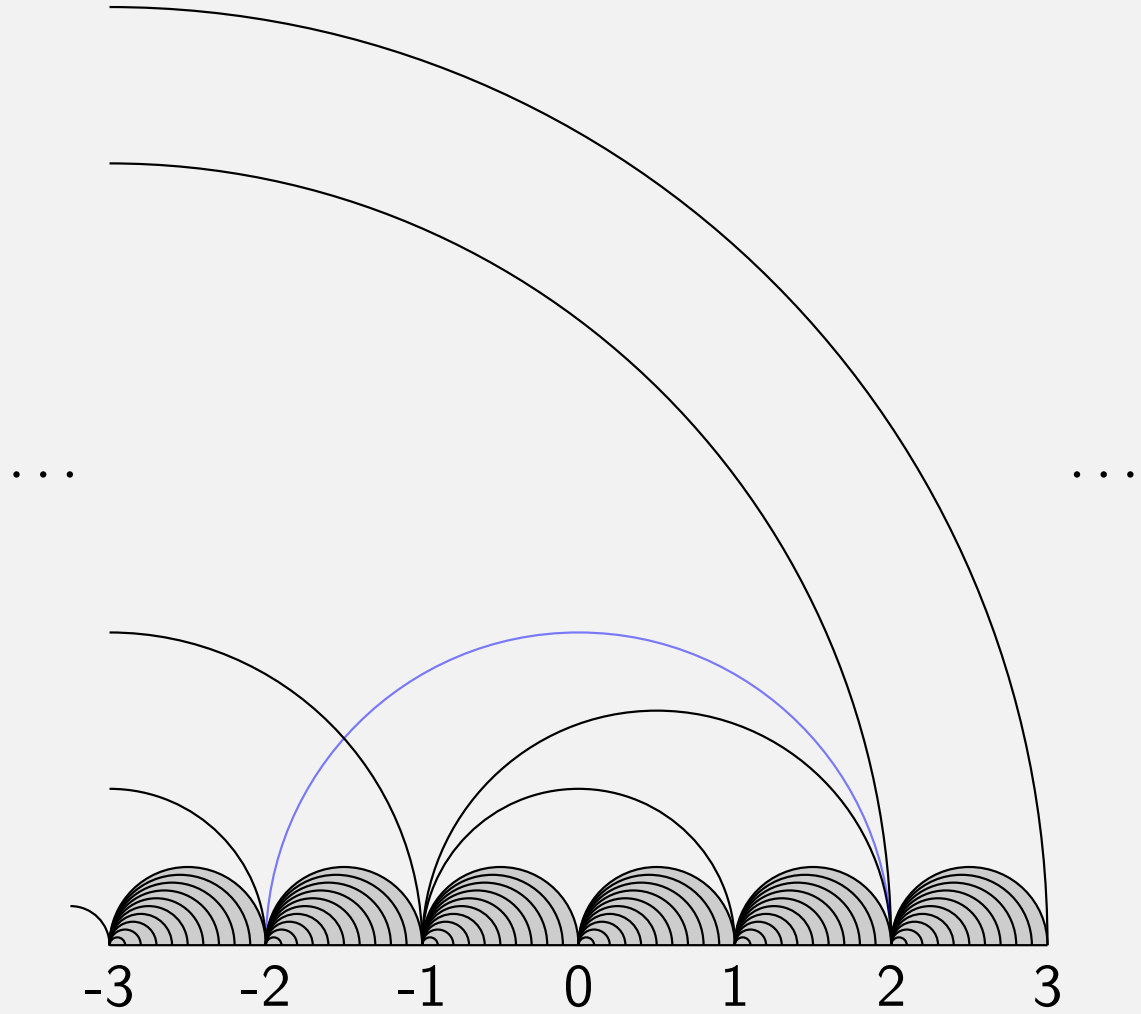

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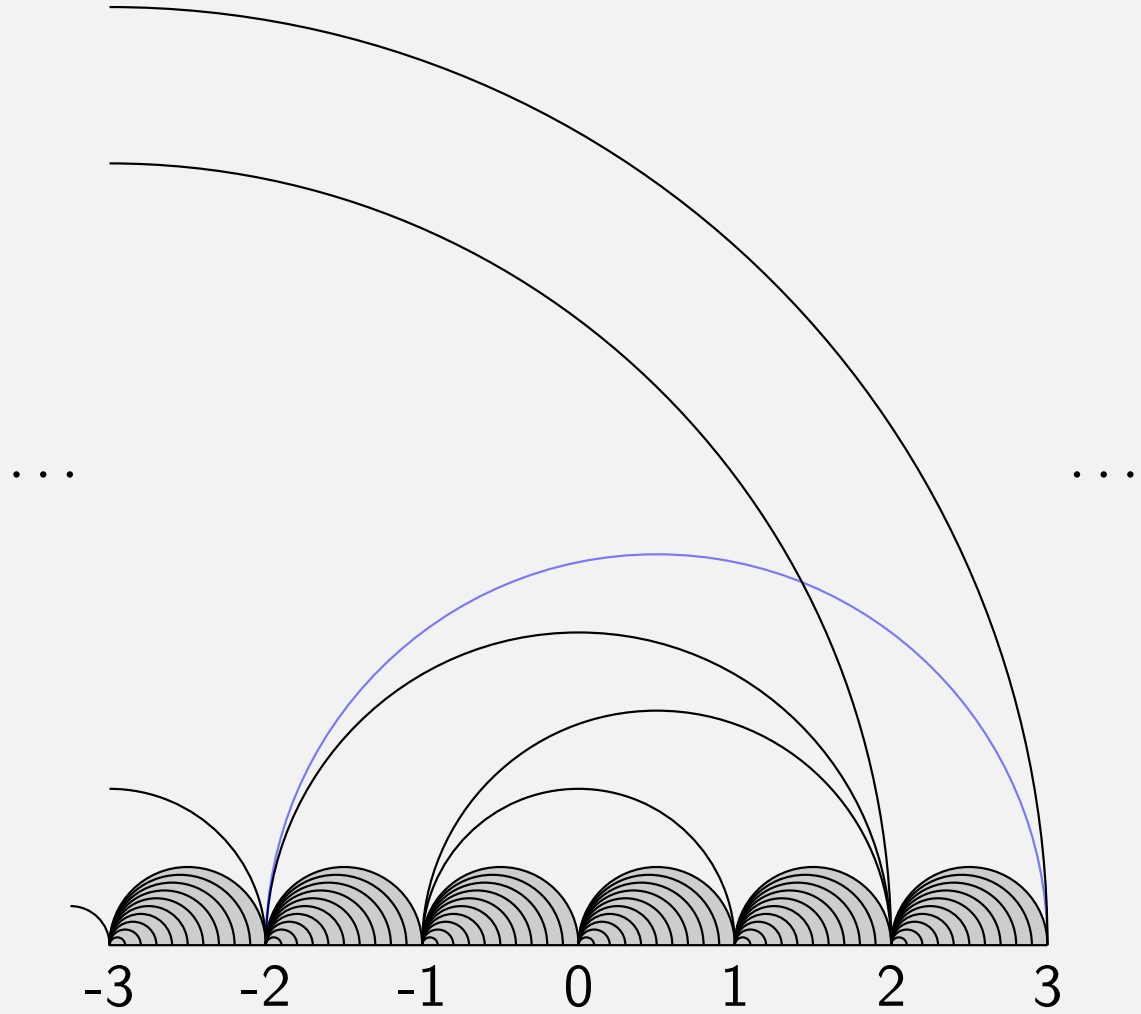

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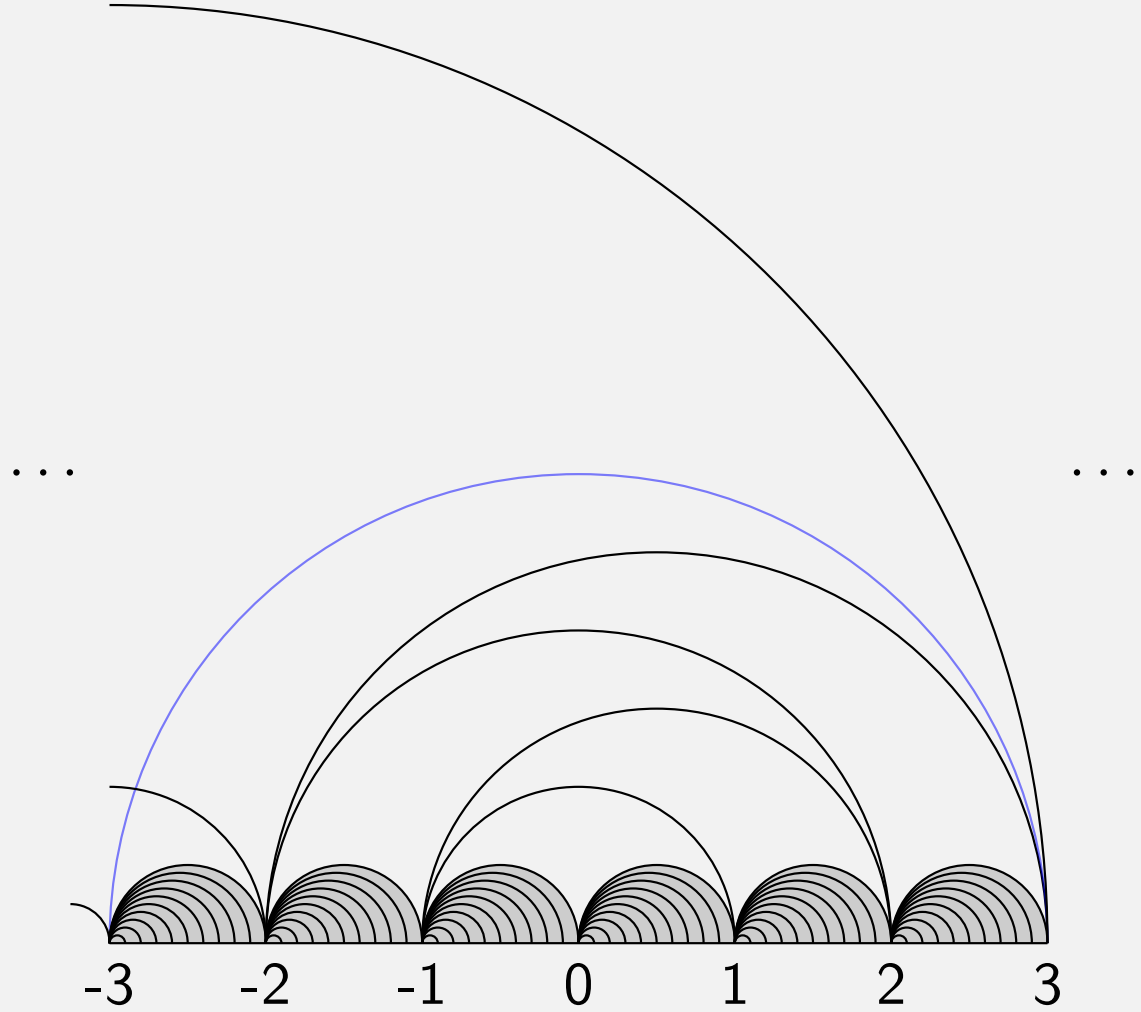

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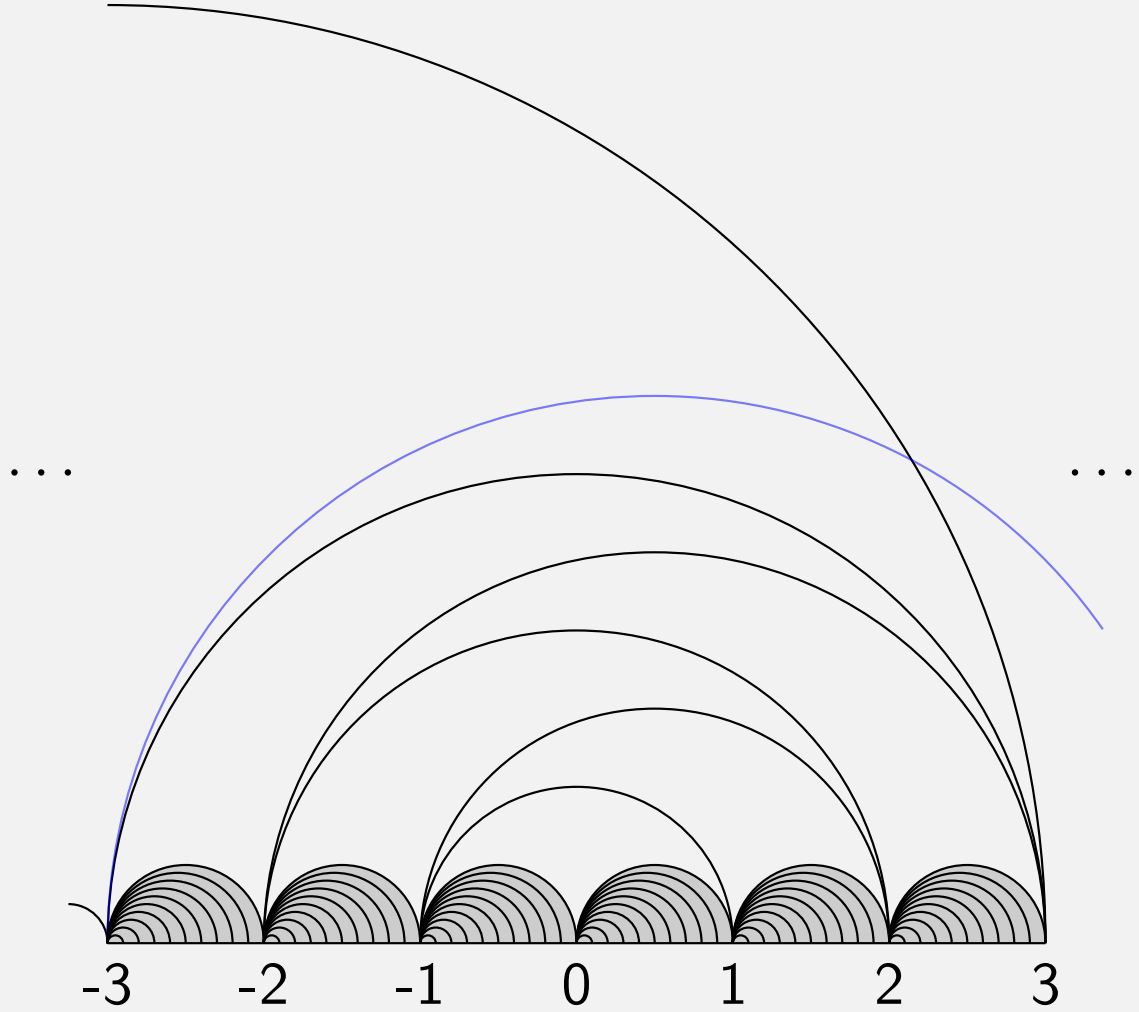

The End

Thank you!