Introduction Semi-invariants Cluster algebras and c-vectors Maximal green sequences Theorems about Maximal green sequences

Semi-invariant pictures, c-vectors, maximal green sequences

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No gap conjecture/theorem

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Maximal green sequences	Derived category
Theorems about Maximal green sequences	Cluster category

These are indecomposable objects in the cluster category. There will be much more information attached to this picture.



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These are the c-vectors.

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Maximal green sequences.

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Quiver Representations

$Q=(Q_0,Q_1) \qquad Q_0 =n$	assume no oriented cycles,	k a field,	kQ = H
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Definition (The category of finitely generated representations of quiver Q)

A k-representation of Q is:

 \square a collection of finite dimensional k-vector spaces: $\{V_i\}_{i\in O_n}$

• a collection of k-linear maps: $\{\varphi_i^V: V_i \to V_i\}_{i \to i \in O_2}$

 $\{j\}$, h morphism $f := \{i\} : V \longrightarrow W$ is a collection of linear maps $\{j\} : V_i \longrightarrow W_i\}_{i \in \mathcal{G}_i}$ so that

 $\| \phi \phi \| = \phi^{2}_{1} \circ h.$

Example (Indecomposable representations of $Q = A_3$: 1 \leftarrow 2 \leftarrow 3



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- a collection of finite dimensional k-vector spaces: {V_i}_{i ∈ Q₀}
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Example (Indecomposable representations of $Q = A_3$: $1 \leftarrow 2 \leftarrow 3$

 $(0 - S_1 := k := 0 := 0$, simple, projective .

 $0: S_2 : 0 \leftarrow k \leftarrow 0$ simple.

 $0: S_2: 0 \leftarrow 0 \leftarrow k$ simple, injective

 $0: P_2 : k \leftarrow k \leftarrow 0$ projective

 $0 \cdot p_1 : k \leftarrow k \leftarrow k$ projective, injective,



Quiver Representations

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Example (Indecomposable representations of $Q = A_3$: $1 \leftarrow 2 \leftarrow 3$

 \bigcirc $S_1: \ k \leftarrow 0 \leftarrow 0$ simple, projective

 $\mathbb{V} = S_2 := 0 \rightarrow -k \leftarrow 0$ simple

 $(0, S_{0}: 0 \leftarrow 0 \leftarrow k)$ simple, injective

 $0 = P_2 := k := -k := -0 : \text{projective}$

 $0 > P_0 : = k \leftarrow k \leftarrow k$ projective, injecti

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- S₁ : $k \leftarrow 0 \leftarrow 0$ simple, projective
- \bigcirc S_2 : 0 \leftarrow k \leftarrow 0 simple
- \bigcirc S_3 : 0 \leftarrow 0 \leftarrow k simple, injective
- $P_2: k \leftarrow k \leftarrow 0 \text{ projective}$
- P₃ : $k \leftarrow k \leftarrow k$ projective, injective
- $\bigcirc I_2 : 0 \leftarrow k \leftarrow k \quad injective$



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$$\bigcirc$$
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$$S_2: 0 \leftarrow k \leftarrow 0$$
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- $P_2: k \leftarrow k \leftarrow 0$ projective
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Semi-invariants Cluster algebras and c-vectors Maximal green sequences Theorems about Maximal green sequences Quiver representations Auslander-Reiten Theory Tilting modules and Mutations Derived category Cluster category

Auslander-Reiten Theory

Auslander-Reiten Theory is a general theory for artin algebras and more.

Definition (Auslander-Reiten quiver - AR quiver)

Example (Auslander-Reiten quiver for the representations of $Q = A_3$: $1 \leftarrow 2 \leftarrow 3$)

AR-quiver



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$ \begin{array}{c} 1 \\ \textbf{(vertices)} \leftrightarrow \{\text{index} \\ \textbf{(arrows)} \leftrightarrow \{\text{irreduced} \\ \end{array} $	ecomposable modules (isomorphism classes) } ucible morphisms }	
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Definition (Auslander-Reiten quiver - AR quiver) ● {vertices} ↔ {indecomposable modules (isomorphism classes)} ② {arrows} ↔ {irreducible morphisms} Example (Auslander-Reiten quiver for the representations of Q = A₀ : 1 ↔ 2 ↔ 3)

AR-quiver



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AR-quiver



Tilting modules

Tilting modules were intensively studied in representation theory of artin algebras in general, and specially of hereditary artin algebras by: Brenner, Butler, Bongartz, Happel, Ungar,

Definition (Tilting module for hereditary algebra H

Definition (Mutation of tilting module)

Mutation ν_i in direction *i* of the tilting module T is a new tilting module $\nu_i T$

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Definition (Tilting module for hereditary algebra H) $Ext^{1}_{H}(T, T) = 0$ $T = T_{1} \oplus \dots \oplus T_{n}$ where $n = |O_{1}|$

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$$\nu_i T = T_1 \oplus \cdots \oplus \boxed{T'_i} \oplus \cdots \oplus T_n.$$

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Examples of Tilting modules and Mutations



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Examples of Tilting modules and Mutations



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Examples of Tilting modules and Mutations



Example (Tilting modules for $Q = A_3 : 1 \leftarrow 2 \leftarrow 3$)
$5 S_3 \oplus P_1 \oplus P_3$

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Example (Tilting modules for $Q = A_3 : 1 \leftarrow 2 \leftarrow 3$)	
$ \begin{array}{c} 1 P_1 \oplus P_2 \oplus P_3 \\ 2 S_2 \oplus P_2 \oplus P_3 \\ 3 S_2 \oplus l_2 \oplus P_3 \\ 4 S_3 \oplus l_2 \oplus P_3 \\ \hline \end{array} $	

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Examples of Tilting modules and Mutations

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Example (Tilting modules for $Q = A_3 : 1 \leftarrow 2 \leftarrow 3$)		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		

Remark

- These are all the tilting modules for the algebra H = IcAq
- The module P_2 could never be replaced.
- in cluster algebras mutations can always b done in ALL directions (wa will see later).

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Examples of Tilting modules and Mutations



Example (Tilting modules for $Q = A_3$: $1 \leftarrow 2 \leftarrow 3$)	
$ \begin{array}{c} 1 \\ P_1 \oplus P_2 \oplus P_3 \\ 2 \\ S_2 \oplus P_2 \oplus P_3 \\ 3 \\ S_2 \oplus P_2 \oplus P_3 \\ 4 \\ S_3 \oplus P_2 \oplus P_3 \\ 5 \\ S_3 \oplus P_1 \oplus P_3 \end{array} $	

Remark		

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2	2 The module P ₃ could never be replaced!	
	We need to consider larger category - we will look at the derived category.	

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Examples of Tilting modules and Mutations



Example (Tilting modules for $Q = A_3 : 1 \leftarrow 2 \leftarrow 3$)
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
$[5] S_3 \oplus P_1 \oplus P_3$

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Examples of Tilting modules and Mutations



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Example (Tilting modules for $Q = A_3 : 1 \leftarrow 2 \leftarrow 3$)	
1 $P_1 \oplus P_2 \oplus P_3$ 2 $S_2 \oplus P_2 \oplus P_3$ 3 $S_2 \oplus I_2 \oplus P_3$ 4 $S_3 \oplus I_2 \oplus P_3$	
$5 \hspace{0.1in} S_3 \oplus P_1 \oplus P_3$	

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1	These are all the tilting modules for the algebra $H = kA_3$
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Derived category $D^{b}(H)$

Since H is hereditary the indecomposable objects in the bounded derived category $D^{b}(H)$ are isomorphic to stalk complexes, so the derived category can be viewed as the union:

 $D^{b}(H) = \bigcup_{i \in \mathbb{Z}} (modH)[i].$

Example (Auslander-Reiten quiver of the derived category of $Q = A_3$: $1 \leftarrow 2 \leftarrow 3$)

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AR-quiver



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Derived category $D^{b}(H)$

Since H is hereditary the indecomposable objects in the bounded derived category $D^{b}(H)$ are isomorphic to stalk complexes, so the derived category can be viewed as the union:

 $D^{b}(H) = \bigcup_{i \in \mathbb{Z}} (modH)[i].$



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Cluster category $C_Q = C_H$

Definition (Cluster category)

Let τ is the Auslander-Reiten translate and [1] is the shift functor on the derived category. Then the cluster category is defined as the orbit category:

 $\mathcal{C}_Q = \mathcal{C}_H := D^b(H)/(\tau^{-1}[1])$

Remark

Cluster category C_O is triangulated category (Keller).

Indecomposable objects in a fundamental domain can be taken as $ind(H) \cup \{P_i[1]\}$.

Example

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Cluster tilting object and Mutations

Definition (Cluster tilting objects)

Edg(1, 1) = 0
 3 = 1, 8 = -8 Jacobies n = 10;

Example (additional in C_Q)

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Cluster tilting object and Mutations

Definition (Cluster tilting objects)

- $Ext^1_{\mathcal{C}_O}(T,T) = 0$
- $T = T_1 \oplus \cdots \oplus T_n$, where $n = |Q_0|$

	Example (additional in C_Q)
Example (in <i>modH</i>)	$ P_1 \oplus P_2[1] \oplus P_3[1] $
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$ S_3 \oplus I_2 \oplus P_3 $	$ S_3 \oplus P_{[1]} \oplus I_2 $
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Semi-invariants on Representation spaces (classical) for $\alpha \in$ Semi-invariants on Presentation spaces for $\alpha \in \mathbb{Z}^n$

Semi-invariants on Representation spaces for $\alpha \in \mathbb{N}^n$

For an acyclic quiver Q with n vertices the theory of semi-invariants on the representation spaces was developed largely by A.King, A. Schofield, H.Derksen, J.Weyman.

Definition (Representation space $Rep(\alpha, Q)$ and Gl(g)-action)

Let $\alpha \in \mathbb{N}^n$. Then the representation space and group action are defined as:

 $\operatorname{Rep}(\alpha, Q) := \prod_{i \to i \in Q_1} \operatorname{Hom}_k(k^{\alpha j}, k^{\alpha i}).$

The group $G := \prod_{i \in Q_n} Gl(\alpha_i)$ acts on $Rep(\alpha, Q)$ as $g\varphi = (g_i)(\varphi_{ji}) := (g_i \circ \varphi_{ji} \circ (g_j)^{-1})$

Definition (Semi-invariants)

A polynomial function $\sigma : \operatorname{Rep}(\alpha, Q) \to k$ is called semi-invariant of weight $\beta = (\beta_i)_{i \in O_0}$ if $\sigma(g\varphi) = \prod_{i \in O_0} (\operatorname{det}(g_i))^{\beta_i} \sigma(\varphi).$

Definition (Domains of semi-invariants)

Let β be a real Schur root. Then $D(\beta) := \{ \alpha \in \mathbb{N}^n | Rep(\alpha, Q) \text{ has a semi-invariant of weight } \beta \}$

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 $\mathbf{G} := \mathbf{G}(\gamma) \times (\mathbf{G}(\delta))^{G}$

where $g \in G(q)$, $h \in G(\delta)$, $\varphi \in Pres(\alpha, \beta)$.

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Gordana Todorov Semi-invariant pictures, c-vectors, maximal green sequences

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Definition (Presentation space $Pres(\alpha, Q)$, action of a group G for $\alpha \in \mathbb{Z}^n$)

 $Pres(\alpha, Q) := Hom_H(P^{(1)}, P^{(0)}),$

where
$$P^{(1)}$$
, $P^{(0)}$ are projective *H*-modules such that $\alpha = \underline{dim}P^{(0)} - \underline{dim}P^{(1)}$,
 $G := Gl(\gamma) \times (Gl(\delta))^{op}$

where
$$P^{(0)} = \coprod P_i^{\gamma_i}$$
 and $P^{(1)} = \coprod P_i^{\delta_i}$, and $G/(\gamma) = \coprod G/(\gamma_i)$ and $G/(\delta) = \coprod G/(\delta_i)$
 $(g, h)\varphi := g \circ \varphi \circ h$,

where $g \in Gl(\gamma), h \in Gl(\delta), \varphi \in Pres(\alpha, Q)$.

Definition (Semi-invariants on $Pres(\alpha, Q)$)

A polynomial function σ : $Pres(\alpha, Q) \rightarrow k$ is called semi-invariant of weight $\beta = (\beta_i)_{i \in Q_0}$ if $\sigma((g, h)\varphi) = \prod_{i \in Q_0} (det(g_i))^{\beta_i} (det(h_i))^{\beta_i} \sigma(\varphi).$
Semi-invariants on Representation spaces (classical) for $\alpha \in$ Semi-invariants on Presentation spaces for $\alpha \in \mathbb{Z}^n$

Domains of semi-invariants for $\alpha \in \mathbb{Z}^n$

Definition (Domains of semi-invariants)

Let β be a real Schur root. Then $D(\beta) := \{\alpha \in \mathbb{Z}^n | Pres(\alpha, Q) \text{ has a semi-invariant of weight } \beta \}.$

Theorem (Stability conditions)

 $egin{aligned} D(eta) &= \{ lpha \in \mathbb{Z}^n | \ &\langle lpha, eta
angle = 0, ig\langle lpha, eta' ig
angle \leq 0 ext{ for } eta' ext{ real Schur subroot of } eta \ \}. \end{aligned}$

Definition

 $\mathcal{D}_{\mathbb{R}}(eta) := \{ lpha \in \mathbb{R}^n | \\ \langle lpha, eta
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Remark (IOTW)

From the joint work: K. Igusa, K. Orr, J. Weyman, G.T.: Modulated semi-invariants. arXiv:1507.03051



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Maximal green sequences Theorems about Maximal green sequences Cluster algebras Cluster Categories and Cluster Algebras

Cluster algebras - Fomin, Zelevinsky

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For each *i* mutate the cluster seed (<u>x</u>, B) using matrix B and get a new cluster seed μ₁(<u>x</u>, B) = (μ₁<u>x</u>, μ₁B).
Continue mutating in all directions, get new cluster variables, new clusters, new cluster seeds.
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Cluster algebras Cluster Categories and Cluster Algebras

Relation between Cluster categories and algebras

Theorem

Let Q be an acyclic quiver and let B be the associated skew-symmetric matrix. Let C_Q be the associated cluster category and A_B the associate cluster algebra. Then there are bijections which respect the cluster structures:

{isomorphism classes of indecomposable rigid objects in C_0 } \leftrightarrow {cluster variables in A_B }

[cluster tilting objects in $\mathcal{C}_{\mathcal{Q}}\} \leftrightarrow \{$ clusters in $\mathcal{A}_{\mathcal{Q}}\}$

 $\{mutations in C_O\} \leftrightarrow \{mutations in A_O\}$

Example (Cluster category and cluster algebra for the quiver $Q = A_3$: 1 \leftarrow 2 \leftarrow 3)

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Cluster category C_O

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- Cluster category C_O
- Initial cluster tilting object $P_1[1] \oplus P_2[1] \oplus P_3[1]$
- Mutation $\nu_1(P_1[1] \oplus P_2[1] \oplus P_3[1]) = P_1 \oplus P_2[1] \oplus P_3[1]$
- Mutation $\nu_2(P_1 \oplus P_2[1] \oplus P_3[1]) = P_1 \oplus P_2 \oplus P_3[1]$

- Cluster algebra \mathcal{A}_Q
- Initial cluster $\{x_1, x_2, x_3\}$
- Mutation
 - $\mu_1\{x_1, x_2, x_3\} = \{\frac{x_2+1}{x_1}, x_2, x_3\}$
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- Cluster algebra A_Q
 Initial cluster {x₁, x₂, x₃}
 - Mutation $\mu_1\{x_1, x_2, x_3\} = \{\frac{x_2+1}{x_1}, x_2, x_3\}$
- Mutation $\mu_2\{\frac{x_2+1}{2}, x_2, x_3\} = \{\frac{x_2+1}{2}, \dots, x_n\}$

Cluster algebras Cluster Categories and Cluster Algebras

Relation between Cluster categories and algebras

Theorem

Let Q be an acyclic quiver and let B be the associated skew-symmetric matrix. Let C_Q be the associated cluster category and A_B the associate cluster algebra. Then there are bijections which respect the cluster structures:

{isomorphism classes of indecomposable rigid objects in C_Q } \leftrightarrow {cluster variables in A_B }

{cluster tilting objects in C_Q } \leftrightarrow {clusters in A_Q }

{mutations in C_Q } \leftrightarrow {mutations in A_Q }

Example (Cluster category and cluster algebra for the quiver $Q = A_3$: $1 \leftarrow 2 \leftarrow 3$)

- Cluster category CQ
- Initial cluster tilting object $P_1[1] \oplus P_2[1] \oplus P_3[1]$
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- Cluster algebra \mathcal{A}_Q
- Initial cluster $\{x_1, x_2, x_3\}$

• Mutation $\mu_1 \{x_1, x_2, x_3\} = \{\frac{x_2+1}{x_1}, x_2, x_3\}$

Mutation

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• Mutation $\mu_{2}\left\{\frac{x_{2}+1}{x_{1}}, x_{2}, x_{3}\right\} = \left\{\frac{x_{2}+1}{x_{1}}, \frac{x_{2}+1+x_{1}x_{3}}{x_{1}x_{2}}, x_{3}\right\}$

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Semi-invariant pictures, c-vectors, maximal green sequences

Theorems about Maximal green sequences

c-vectors

c-vectors Cluster tilting objects, domains of semi-invariants and c-vector Green mutation



Theorem (Sign coherence - Nakanishi-Zelevinsky

The c-vectors are sign coherent, i.e. each vector has all nonnegative entries or all nonpositive entries.

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Introduction Semi-invariants Cluster algebras and c-vectors Maximal green sequences

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Theorem (IOTW) After a sequence of mutations, the c-vectors corresponding to the cluster tilting object T are given by the labels of the weights of semi-invariants which define the walls $D(\beta)$ of the new simplex with vertices { $T_1, ..., T_n$ } and the sign can be read of the semi-invariant

picture as indicated by the normal orientation (concavity).

Example

$$T = P_1 \oplus P_2[1] \oplus P_3[1]$$

$$c_1 = -[100], c_2 = +[010], c_3 = +[001]$$



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Gordana Todorov Semi-invariant pictures, c-vectors, maximal green sequences

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Maximal green sequences



Definition (green mutation)

Mutation in the direction of a nonnegative c-vector is called green mutation.

Definition (Maximal green sequence)

A finite sequence of green mutations, starting from the initial cluster tilting object $P_1[1] \oplus ... \oplus P_n[1]$ which cannot be extended by another green mutation is called a maximal green sequence.

Remark

A maximal green sequence, if it exists, it must stop in the simplex whose vertices are projective modules, i.e. cluster tilting object $P_1 \oplus \ldots \oplus P_n$.

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Remark



Sink before source theorem Finiteness theorem No gap conjecture/theorem

Sink before source Theorem

Theorem (T. Brustle, S. Hermes, K. Igusa, GT)

Suppose $\#{j \rightarrow i} \ge 2$. Then every maximal green sequence must mutate at sink i before it mutates at the source j".

Theorem (General, BHIT)

For each arrow of infinite representation type, every maximal green sequence must mutate at the sink before the source.

Remark

The idea of the proof is coming from this picture since maximal green sequence cannot cross infinitely many walls.

arXiv:1503.07945



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cannot be completed maximal green sequence $Q: j \Rightarrow i \rightarrow b$

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Finiteness theorem

Theorem (BHIT)

If Q is a quiver which is mutation equivalent to an acyclic tame quiver then Q has at most finitely many maximal green sequences.

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No gap conjecture/theorem

Conjecture

Let Q be an acyclic quiver. Suppose there are maximal green sequences of lengths m_1 and m_2 . Then for every integer between m_1 and m_2 there exists a maximal green sequence of that length.

Theorem (Hermes, Igusa)

Let Q be a simply laced acyclic quiver of tame type. Suppose there are maximal green sequences of lengths m_1 and m_2 . Then for every integer between m_1 and m_2 there exists a maximal green sequence of that length.

Remark

No gap conjecture is false for valued quivers. For example the quiver B_2 has maximal green sequences of lengths 2 and 4 and none of length 3.

Remark

Conjecture is still open in wild case.

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