# Semi-invariant pictures, c-vectors, maximal green sequences

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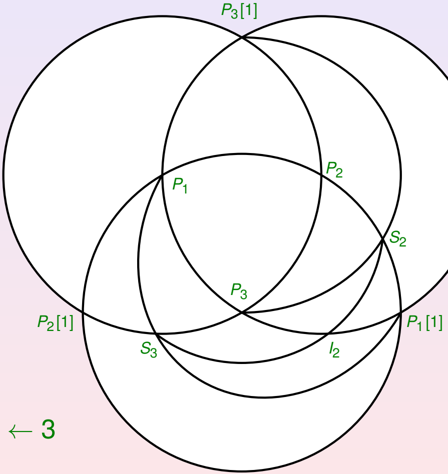
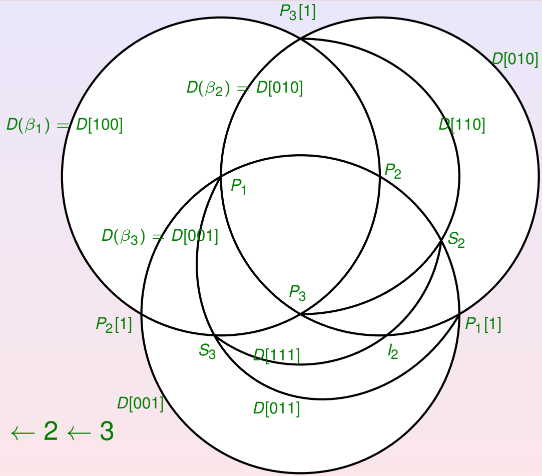
# Table of contents

These are indecomposable objects in the cluster category. There will be much more information attached to this picture.

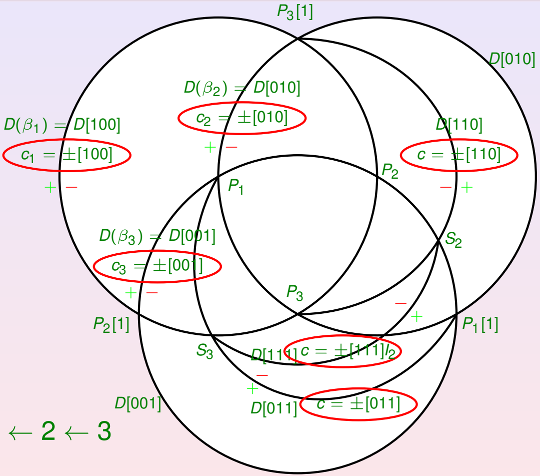



$$Q = 1 \leftarrow 2 \leftarrow 3$$



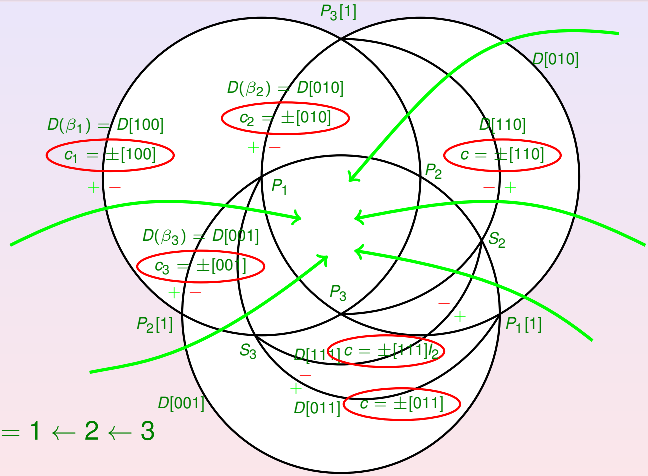

$$Q = 1 \leftarrow 2 \leftarrow 3$$

Domains of semi-invariants.




$$Q = 1 \leftarrow 2 \leftarrow 3$$

These are the c-vectors.




$$Q = 1 \leftarrow 2 \leftarrow 3$$

Maximal green sequences.

# Quiver Representations

$Q = (Q_0, Q_1)$ $|Q_0| = n$ assume no oriented cycles, $k$ a field, $kQ = H$

**Definition (The category of finitely generated representations of quiver Q)**

1. A $k$-representation of $Q$ is:
   - a collection of finite dimensional $k$-vector spaces:
   - a collection of $k$-linear maps:

**Example ( Indecomposable representations of $Q = A_3 : \; 1 \leftarrow 2 \leftarrow 3$ )**

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2. A morphism $f = (f_i) : V \to W$ is a collection of linear maps $\{f_i : V_i \to W_i\}_{i \in Q_0}$ so that $f_j \circ \varphi_{ji}^V = \varphi_{ji}^W \circ f_i$.

**Example ( Indecomposable representations of $Q = A_3 : 1 \leftarrow 2 \leftarrow 3$ )**

- $S_1 :$ $k \leftarrow 0 \leftarrow 0$ simple, projective
- $S_2 :$ $0 \leftarrow k \leftarrow 0$ simple
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# Auslander-Reiten Theory

Auslander-Reiten Theory is a general theory for artin algebras and more.

**Definition (Auslander-Reiten quiver - AR quiver)**

1. {vertices} $\leftrightarrow$ {indecomposable modules (isomorphism classes)}
2. {arrows} $\leftrightarrow$ {irreducible morphisms}

**Example (Auslander-Reiten quiver for the representations of $Q = A_3 : \ 1 \leftarrow 2 \leftarrow 3$ )**

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# Tilting modules

Tilting modules were intensively studied in representation theory of artin algebras in general, and specially of hereditary artin algebras by: Brenner, Butler, Bongartz, Happel, Ungar, ....

**Definition (Tilting module for hereditary algebra H)**

1. $Ext(T,T) = 0$
2. $T = T_1 \oplus \cdots \oplus T_n$ where $n = |Q_0|$

**Definition (Mutation of tilting module)**

Mutation $\nu_i$ in direction $i$ of the tilting module $T$ is a new tilting module $\nu_i T$:

$$T = T_1 \oplus \cdots \oplus \boxed{T_i} \oplus \cdots \oplus T_n$$

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# Examples of Tilting modules and Mutations

**Example (Auslander-Reiten quiver for the representations of $Q = A_3 : \ 1 \leftarrow 2 \leftarrow 3$)**

AR-quiver

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AR-quiver

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# Derived category $D^b(H)$

Since $H$ is hereditary the indecomposable objects in the bounded derived category $D^b(H)$ are isomorphic to stalk complexes, so the derived category can be viewed as the union:

$$D^b(H) = \bigcup_{i\in\mathbb{Z}}(modH)[i].$$

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Let $\tau$ is the Auslander-Reiten translate and $[1]$ is the shift functor on the derived category. Then the cluster category is defined as the orbit category:

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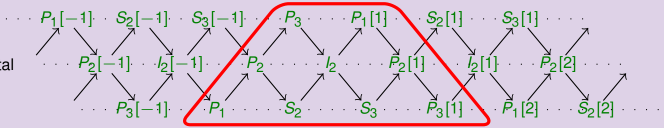
## Example

Fundamental Domain



# Cluster tilting object and Mutations

**Definition (Cluster tilting objects)**

**Example (in *modH*)**

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# Cluster tilting object and Mutations

**Definition (Cluster tilting objects)**

- $Ext^1_{\mathcal{C}_Q}(T, T) = 0$
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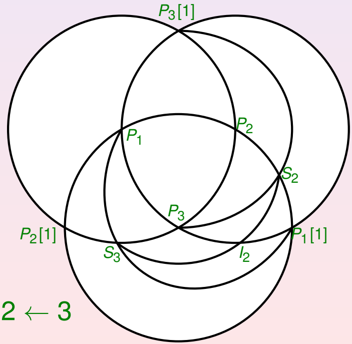


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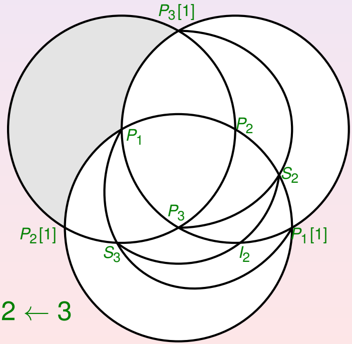


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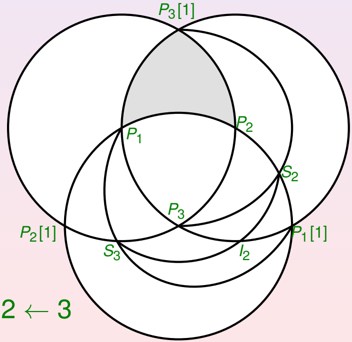


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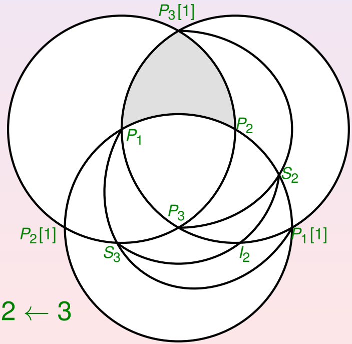


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# Semi-invariants on Representation spaces for $\alpha \in \mathbb{N}^n$

For an acyclic quiver $Q$ with $n$ vertices the theory of semi-invariants on the representation spaces was developed largely by A.King, A. Schofield, H.Derksen, J.Weyman.

**Definition (Representation space $Rep(\alpha, Q)$ and $Gl(g)$-action)**

Let $\alpha \in \mathbb{N}^n$. Then the representation space and group action are defined as:

$$Rep(\alpha, Q) := \prod_{j \to i \in Q_1} Hom_k(k^{\alpha_j}, k^{\alpha_i}).$$

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For an acyclic quiver $Q$ with $n$ vertices the theory of semi-invariants on the representation spaces was developed largely by A.King, A. Schofield, H.Derksen, J.Weyman.

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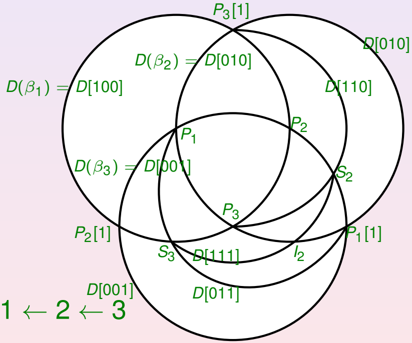
From the joint work: K. Igusa, K. Orr, J. Weyman, G.T.: Modulated semi-invariants. arXiv:1507.03051

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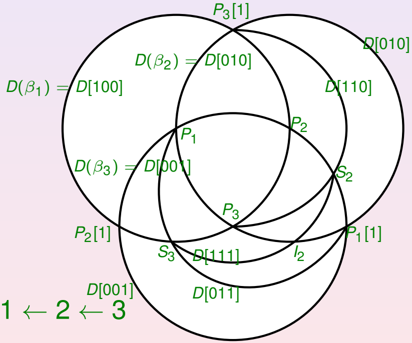


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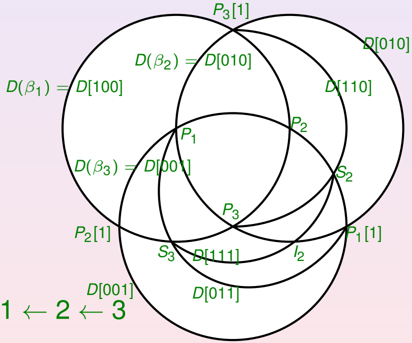


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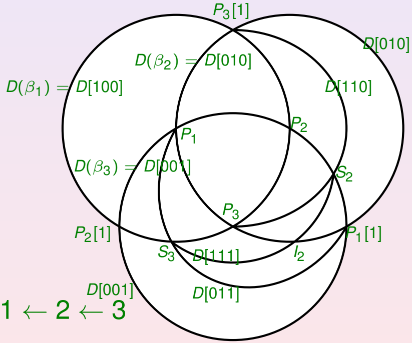


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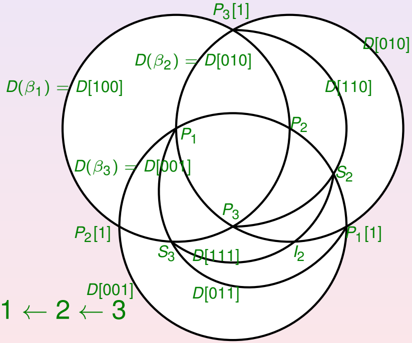


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# Cluster algebras - Fomin, Zelevinsky

**Remark ( Acyclic quivers $Q$ $\leftrightarrow$ skew symmetric matrices $B$ )**

To an acyclic quiver $Q$ associate a skew-symmetric matrix $B$ as $B = E^t - E$ where $E$ is the Euler matrix, i.e.
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**Example ($Q = A_3$ : $1 \leftarrow 2 \leftarrow 3$)**

$$(1 \leftarrow 2 \leftarrow 3) \quad \leftrightarrow \quad \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

**Definition (Cluster algebra)**

Let $B$ be a $n \times n$ skew-symmetric matrix. The cluster algebra $\mathcal{A}_B$ is generated by cluster variables as a subalgebra of the field of rational functions in $n$ variables.

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To an acyclic quiver $Q$ associate a skew-symmetric matrix $B$ as $B = E^t - E$ where $E$ is the Euler matrix, i.e. $E_{ij} = dim_k Hom_H(S_i, S_j) - dim_k Ext^1_H(S_i, S_j)$.

**Example** ($Q = A_3$ : $1 \leftarrow 2 \leftarrow 3$)

$$(1 \leftarrow 2 \leftarrow 3) \quad \leftrightarrow \quad \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

**Definition** (Cluster algebra)

Let $B$ be a $n \times n$ skew-symmetric matrix. The cluster algebra $\mathcal{A}_B$ is generated by cluster variables as a subalgebra of the field of rational functions in $n$ variables.

- Initial cluster is a transcendence basis: $\underline{x} = \{x_1, .., x_n\}$. Initial cluster seed is the pair $(\underline{x}, B)$.
- For each $i$ mutate the cluster seed $(\underline{x}, B)$ using matrix $B$ and get a new cluster seed $\mu_i(\underline{x}, B) = (\mu_i \underline{x}, \mu_i B)$.
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# Relation between Cluster categories and algebras

**Theorem**

*Let $Q$ be an acyclic quiver and let $B$ be the associated skew-symmetric matrix. Let $\mathcal{C}_Q$ be the associated cluster category and $\mathcal{A}_B$ the associate cluster algebra. Then there are bijections which respect the cluster structures:*

$$\{\text{isomorphism classes of indecomposable rigid objects in } \mathcal{C}_Q\} \leftrightarrow \{\text{cluster variables in } \mathcal{A}_B\}$$

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# c-vectors

**Definition**

- Let $B$ be a skew symmetric matrix. Define the extended matrix $\tilde{B} := \begin{bmatrix} B \\ I_n \end{bmatrix}$ where $I_n$ is the identity matrix.
- Perform a mutation on the extended matrix $\tilde{B}$ and obtain a new matrix $\mu_i \tilde{B} := \begin{bmatrix} \mu_i B \\ \mu_i I_n \end{bmatrix}$.
- Perform sequences of mutations and obtain new matrices $\begin{bmatrix} B' \\ C' \end{bmatrix}$
- c- matrices are the matrices that appear in the lower half of these $2n \times n$ matrices.
- c-vectors are the column vectors of c-matrices.

**Theorem (Sign coherence - Nakanishi-Zelevinsky)**

*The c-vectors are sign coherent, i.e. each vector has all nonnegative entries or all nonpositive entries.*

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- Perform sequences of mutations and obtain new matrices $\begin{bmatrix} B' \\ C' \end{bmatrix}$
- c- matrices are the matrices that appear in the lower half of these $2n \times n$ matrices.
- c-vectors are the column vectors of c-matrices.

## Theorem (Sign coherence - Nakanishi-Zelevinsky)

*The c-vectors are sign coherent, i.e. each vector has all nonnegative entries or all nonpositive entries.*

# Cluster tilting objects, domains of semi-invariants and c-vectors

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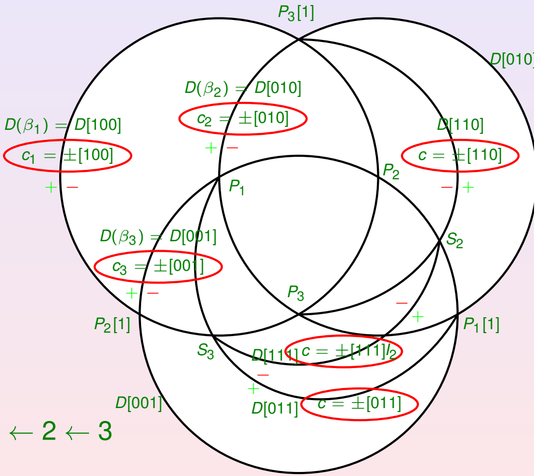



**Theorem (IOTW)**

*After a sequence of mutations, the $c$-vectors corresponding to the cluster tilting object $T$ are given by the labels of the weights of semi-invariants which define the walls $D(\beta)$ of the new simplex with vertices $\{T_1, .., T_n\}$ and the sign can be read of the semi-invariant picture as indicated by the normal orientation (concavity).*

**Example**

$T = P_1 \oplus P_2[1] \oplus P_3[1]$
$c_1 = -[100]$, $c_2 = +[010]$, $c_3 = +[001]$

$Q = 1 \leftarrow 2 \leftarrow 3$

# Maximal green sequences

**Definition ( green mutation )**

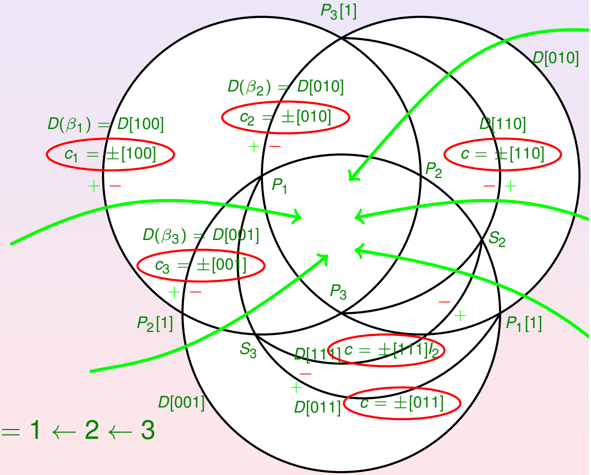
Mutation in the direction of a nonnegative c-vector is called green mutation.

**Definition (Maximal green sequence)**

A finite sequence of green mutations, starting from the initial cluster tilting object $P_1[1] \oplus .. \oplus P_n[1]$ which cannot be extended by another green mutation is called a maximal green sequence.

**Remark**

A maximal green sequence, if it exists, it must stop in the simplex whose vertices are projective modules, i.e. cluster tilting object $P_1 \oplus .. \oplus P_n$.

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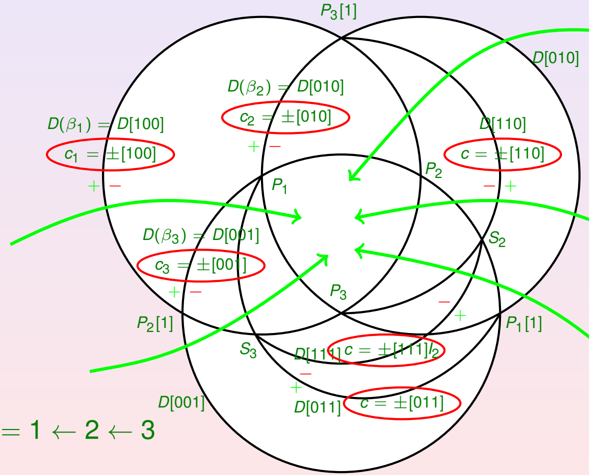


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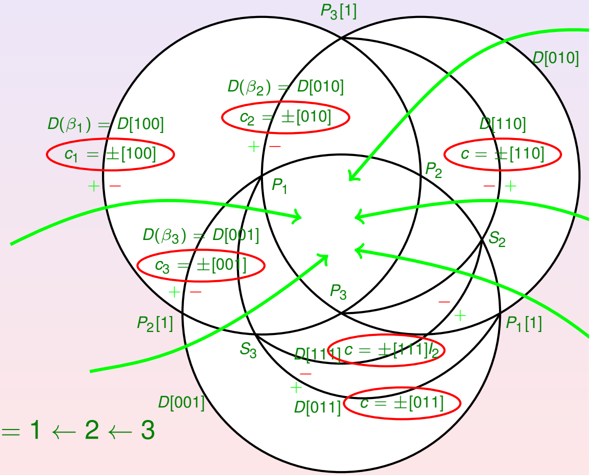


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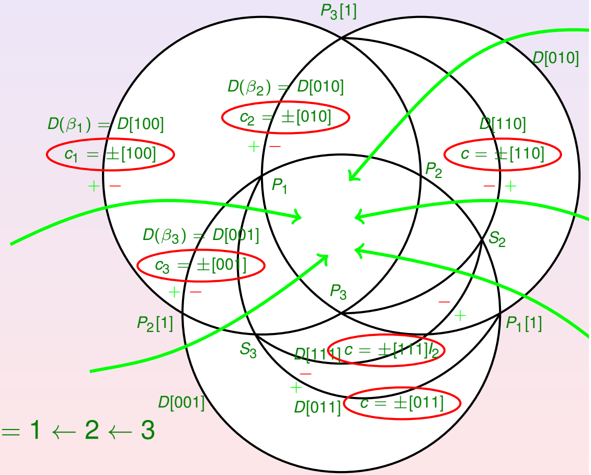


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# Sink before source Theorem

**Theorem (T. Brustle, S. Hermes, K. Igusa, GT)**

*Suppose $\#\{j \to i\} \geq 2$. Then every maximal green sequence must mutate at sink $i$ before it mutates at the source $j$".*

**Theorem (General, BHIT)**

*For each arrow of infinite representation type, every maximal green sequence must mutate at the sink before the source.*

**Remark**

The idea of the proof is coming from this picture since maximal green sequence cannot cross infinitely many walls.

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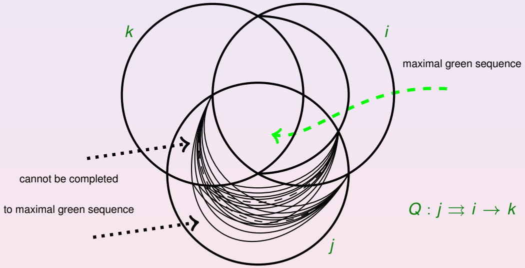

# Finiteness theorem

**Theorem (BHIT)**

*If $Q$ is a quiver which is mutation equivalent to an acyclic tame quiver then $Q$ has at most finitely many maximal green sequences.*

# No gap conjecture/theorem

**Conjecture**

*Let $Q$ be an acyclic quiver. Suppose there are maximal green sequences of lengths $m_1$ and $m_2$. Then for every integer between $m_1$ and $m_2$ there exists a maximal green sequence of that length.*

**Theorem (Hermes, Igusa)**

*Let $Q$ be a simply laced acyclic quiver of tame type. Suppose there are maximal green sequences of lengths $m_1$ and $m_2$. Then for every integer between $m_1$ and $m_2$ there exists a maximal green sequence of that length.*

**Remark**

No gap conjecture is false for valued quivers. For example the quiver $B_2$ has maximal green sequences of lengths 2 and 4 and none of length 3.

**Remark**

Conjecture is still open in wild case.