

# Projective dimension of modules over cluster-tilted algebras

Gordana Todorov, Northeastern University, Boston  
Representation Theory Seminar, Northeastern University  
October 28, 2011

**Abstract:** We study the projective dimension of finitely generated modules over cluster-tilted algebras, i.e. algebras  $\text{End}_{\mathcal{C}}(T)$  where  $T$  is a cluster tilting object in a cluster category  $\mathcal{C}$ .

It is well-known that all finitely generated  $\text{End}_{\mathcal{C}}(T)$ -modules are of the form  $\text{Hom}_{\mathcal{C}}(T, M)$  for objects  $M$  in  $\mathcal{C}$ , and since  $\text{End}_{\mathcal{C}}(T)$  is Gorenstein of dimension 1, the projective dimension of  $\text{Hom}_{\mathcal{C}}(T, M)$  is either zero, one or infinity.

We define the ideal  $I_M$  of  $\text{End}_{\mathcal{C}}(T[1])$  as the ideal of all endomorphisms that factor through  $M$ . We show that the  $\text{End}_{\mathcal{C}}(T)$ -module  $\text{Hom}_{\mathcal{C}}(T, M)$  has projective dimension  $\leq 1$  precisely when the ideal  $I_M$  is zero. Moreover, we apply these results to characterize the location of modules of infinite projective dimension in the Auslander-Reiten quiver.

(This is joint work with Louis Beaudet and Thomas Brüstle.)