GENERA AND CANCELLATION IN STABLE HOMOTOPY CATEGORY

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We consider the stable homotopy category $\mathscr{S}$ of *polyhedra*, i.e. finite cell complexes, and its localizations $\mathscr{S}_p$. Two polyhedra X, Y are said to be *in the same genus* if $X_p \simeq Y_p$ for all prime p. Then we write $X \sim Y$. We prove the following results:

(1) $X \sim Y$ if and only if $X \vee B \simeq Y \vee B$, where B is the wedge of all spheres S^n such that the stable homotopy group $\pi_n^S(X)$ is not finite (there are finitely many of them).

(2) If $X \sim Y$, then $mX \sim mY$ for some m, where mX denotes the wedge of m copies of X.

(3) If $X \vee Z \simeq Y \vee Z$, where $Z \in \operatorname{add} X$, then $X \simeq Y$.

(All isomorphisms are *stable*, i.e. isomorphisms in $\mathscr{S}$.)

The main tool is a one-to-one correspondence of polyhedra $Y \sim X$ and Λ-modules M *of the principal genus*, i.e. such that $M_p \simeq \Lambda_p$ for all p, where Λ is the ring of stable endomorphisms of X.