GENERICALLY TAME ALGEBRAS OVER PERFECT FIELDS

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Let A be a finite-dimensional algebra over a field k.

Definitions (W. Crawley-Boevey) A left A-module G is called generic if it is indecomposable of finite length over $\mathrm{End}_A(G)$, but of infinite dimension over k.

A finite-dimensional k-algebra is said to be generically tame if for each natural number d, there is only a finite number of isomorphism classes of indecomposable generic A-modules of endolength equal to d.

Consider the following list of hereditary algebras B:

(1) B is a skew polynomial algebra $D[x, \alpha]$, where D is a finite-dimensional division k-algebra and α is some automorphism of D.

(2) B is the matrix algebra $\begin{pmatrix} F & 0 \\ M & G \end{pmatrix}$, where F and G are finite-dimensional division k-algebras and M is a simple G-F-bimodule where the field k acts centrally. Moreover $\mathrm{dim}_G M = 2 = \mathrm{dim} M_F$.

(3) B is the matrix algebra $\begin{pmatrix} F & 0 \\ M & G \end{pmatrix}$, where F and G are finite-dimensional division k-algebras and M is a simple G-F.bimodule where the field k acts centrally. Moreover, $\mathrm{dim}_G M = 4$ and $\mathrm{dim} M_F = 1$ or $\mathrm{dim}_G M = 1$ and $\mathrm{dim} M_F = 4$.

In the talk I will give an idea of how using Bocses the following result is proved.

Theorem (R.Bautista, E. Pérez, L. Salmerón) Let A be a finite dimensional algebra over a perfect field k which is generically tame, and let d be a natural number. Then there is finite sequence $\{B_i, Z_i\}_{i=1}^n$, where B_i is one of the hereditary algebras listed above and Z_i is an A-B_i-bimodule, which is finitely generated as right B_i-module, satisfying the following:

(1) The functor $Z_i \otimes_{B_i} - : B_i\text{-Mod} \to A\text{-Mod}$ preserves indecomposability and isomorphism classes, of non-injective modules, if B_i is a hereditary algebra of one of the types 2 or 3;

(2) The functor $Z_i \otimes_{B_i} - : B_i\text{-Mod} \to A\text{-Mod}$ preserves indecomposability and isomorphism classes, if B_i is a hereditary algebra of type 1;

(3) Almost every indecomposable A-module M with $\mathrm{dim}_k M \leq d$ is isomorphic to $Z_i \otimes_{B_i} N$ for some i and some B_i-module N.