Continuous cluster categories

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We construct a continuous cluster category which is isomorphic to a limit of finite spaced-out cluster categories and study its basic properties. This continuous cluster category is a K-category which is homogeneous and triangulated by construction and the isomorphism classes of indecomposable objects in the category are in bijection with the points on the open Moebius band. By homogeneous we mean that there are triangulated automorphisms of the category which carry one indecomposable object to any other.

The triangulated structure comes from the fact that this category is an orbit category of a quotient of an exact category having a structure similar to a Frobenius category. Distinguished triangles can be interpreted as immersed ideal polyhedra in the hyperbolic plane with carefully chosen coefficients in K at crossings of sides and ideal vertices. (See example below.) Clusters in the category are defined to be maximal discrete sets of pairwise compatible indecomposable objects.

We use the algebra of the exact category, the topology of the Moebius band and the geometry of the hyperbolic plane to give descriptions of the distinguished triangles. We also discuss the relation to the unbounded cluster category for once punctured surfaces.

This is joint work with Gordana Todorov.



FIGURE 1. If abc = 1, ced = auv = -1 this is a distinguished triangle

$$A \xrightarrow{\binom{e}{a}} B_1 \oplus B_2 \xrightarrow{\binom{d}{b}} C_1 \oplus C_2 \xrightarrow{(c,v)} A[1]$$