

Matrix Problems Associated to Some Brauer Configuration Algebras Maurice Auslander Distinguished Lectures Falmouth-USA

Agustín Moreno Cañadas jointly with; Pedro Fernández, José A. Velez-Marulanda, Hernán Giraldo

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- 2 Brauer Configuration Algebras
- Some Matrix Problems
 The Kronecker Problem

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- Helices and Exceptional Sequences
- Cycles
- The Four Subspace Problem (FSP)

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Bijections between solutions of the Kronecker problem and the four subspace problem with indecomposable projective modules over some Brauer configuration algebras are obtained by interpreting elements of some integer sequences as polygons of suitable Brauer configurations.

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Ideas from the Medellin CIMPA School

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Brauer Configuration Algebras

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Definition

Recently, E.L. Green and S. Schroll introduced Brauer configuration algebras as a way to deal with research of algebras of wild representation type (*Brauer configuration algebras: A generalization of Brauer graph algebras, E.L. Green, S. Schroll, Bull. Sci. Math. vol.* 141, 2017, 539-572).

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A Brauer configuration is a tuple $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathcal{O})$ where Γ_0 is a set of vertices, Γ_1 is a set of polygons, $\mu : \Gamma_0 \to \mathbb{N}$ is a multiplicity function and \mathcal{O} is an orientation, such that the following conditions hold:

C(1) Every vertex in Γ_0 is a vertex in at least one polygon in Γ_1 .

C(3) Every polygon has at least a vertex happening more than once (nontruncated vertex).

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The cyclic ordering at vertex α is obtained by linearly ordering the list (i.e., $V_{i_1} < \cdots < V_{i_t}$ and by adding $V_{i_t} < V_{i_1}$). Such a list is said to be the successor sequence at α .

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The Quiver of a Brauer Configuration Algebra

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The quiver Q_{Γ} of a Brauer configuration algebra is defined in such a way that the vertex set $\{v_1, v_2, \ldots, v_m\}$ of Q_{Γ} is in correspondence with the set of polygons $\{V_1, V_2, \ldots, V_m\}$ in Γ_1 , noting that there is one vertex in Q_{Γ} for every polygon in Γ_1 .

Arrows in Q_{Γ} are defined by the successor sequences.

For each non-truncated vertex $\alpha \in \Gamma_0$ and each successor V' of V at α , there is an arrow from v to v' in Q_{Γ} where v and v' are the vertices in Q_{Γ} associated to the polygons V and V' in Γ_1 , respectively.

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- $\ \, {\bf I}_0=\{1,2,3,4\},$
- **2** $\Gamma_1 = \{U = \{1, 1, 2, 3, 3, 4\}, V = \{1, 2, 3, 4, 4, 4\}\},\$
- ${igsidentsigma}$ At vertex 1, it holds that; U < U < V, val(1) = 3,
- ④ At vertex 2, it holds that; U < V, val(2) = 2,
- (1) At vertex 3, it holds that; U < U < V, val(3) = 3
- At vertex 4, it holds that; U < V < V < V, val(4) = 4,
- $\bigcirc \mu(\alpha) = 1$ for any vertex α .

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$$\mu(\alpha) = 1$$
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Some Properties of Brauer Configuration Algebras

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Definition

Let k be a field and Γ a Brauer configuration. The Brauer configuration algebra associated to Γ is defined to be kQ_{Γ}/I_{Γ} , where Q_{Γ} is the quiver associated to Γ and I_{Γ} is the ideal in kQ_{Γ} generated by a set of relations ρ_{Γ} of type I, II and III.

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The ideal of relations I_{Γ} of the Brauer configuration algebra associated to the Brauer configuration Γ is generated by three types of relations:

1 Relations of type I. For each polygon

 $V = \{\alpha_1, \ldots, \alpha_m\} \in \Gamma_1$ and each pair of non-truncated vertices α_i and α_j in V, the set of relations ρ_{Γ} contains all relations of the form $C^{\mu(\alpha_i)} - C'^{\mu(\alpha_j)}$ where C is a special α_i -cycle and C' is a special α_i -cycle.

- **Relations of type II**. Relations of type II are all paths of the form C^{μ(α)}a where C is a special α-cycle and a is the first arrow in C.
- **Relations of type III**. These relations are quadratic monomial relations of the form *ab* in *kQ*_Γ where *ab* is not a subpath of any special cycle unless *a* = *b* and *a* is a loop associated to a vertex of valency 1 and μ(α) > 1.

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Theorem

Let Λ be a Brauer configuration algebra with Brauer configuration Γ .

- There is a bijective correspondence between the set of projective indecomposable Λ-modules and the polygons in Γ.
- If P is a projective indecomposable Λ-module corresponding to a polygon V in Γ. Then rad P is a sum of r indecomposable uniserial modules, where r is the number of (non-truncated) vertices of V and where the intersection of any two of the uniserial modules is a simple Λ-module.
- ③ A Brauer configuration algebra is a multiserial algebra.
- The number of summands in the heart of a projective indecomposable Λ -module P such that $\operatorname{rad}^2 P \neq 0$ equals the number of non-truncated vertices of the polygons in Γ

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- The number of summands in the heart of a projective indecomposable Λ-module P such that rad² P ≠ 0 equals the number of non-truncated vertices of the polygons in Γ

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Proposition

Let Λ be the Brauer configuration algebra associated to the Brauer configuration Γ . For each $V \in \Gamma_1$ choose a non-truncated vertex α and exactly one special α -cycle C_V at V then

 $\{\overline{p} \mid p \text{ is a proper prefix of some } C^{\mu(\alpha)} \text{ where } C \text{ is a special } \alpha - cycle} \bigcup \{\overline{C^{\mu(\alpha)}} \mid V \in \Gamma_1\} \text{ is a } k\text{-basis of } \Lambda.$

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The Kronecker Problem				

The Kronecker Problem

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The Kronecker Problem					

The classification of indecomposable Kronecker modules was solved by Weierstrass in 1867 for some particular cases and by Kronecker in 1890 for the complex number field case.

This flat matrix problem of type Gelfand is equivalent to the problem of finding canonical Jordan form of pairs (A, B) of matrices with respect to the following elementary transformations:

(i) All elementary transformations on rows of the block matrix (A, B).

(ii) All elementary transformations made simultaneously on columns of *A* and *B* having the same index number.

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The Kronecker Problem				

If k is an algebraically closed field then up to isomorphism every indecomposable Kronecker module belongs to one of the following three classes:

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The Kronecker Problem				
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where F_n is a Frobenius matrix or companion matrix of a minimal polynomial $p^s(t)$ with $n = s \partial p(t)$, $\partial p(t)$ denotes the degree of the polynomial p(t).



where $J_n(0) \in \{J_n^+(0), J_n^-(0)\}$ and $J_n^{\pm}(0)$ denotes a corresponding upper or lower Jordan block. Whereas, I* denotes the dual case defined by the classification problem.



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Pedro Fernández, Hernán Giraldo and A.M.C associated to each indecomposable preprojective Kronecker module some helices which are paths running through the rows of the matrix block as follows:

 $\{a_{1,j}, b_{1,1}, b_{r_1,1}, a_{r_1,s_1}, a_{r_2,s_1}, b_{r_2,s_2}, b_{r_3,s_2}, a_{r_3,s_3}, \dots, l_{r_t,s_t}\}$ where starting vertices are entries in the null row of matrix A.

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Figure: Preprojective (5, 4); A052558={4, 12, 48, 72, ... } (the number of helices associated to a preprojective Kronecker module equals the number of ways of connecting n + 1 equally spaced points on a circle with a path of n line segments ignoring reflections).

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Regarding the number of helices associated to preprojective Kronecker modules, we have the following result (Pedro Fernández, Hernán Giraldo, A.M.C)

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Theorem

If (n + 1, n) denotes an indecomposable preprojective Kronecker module then the number of helices associated to (n + 1, n) is $h_n^p = n! \lceil \frac{n}{2} \rceil$ where $\lceil x \rceil$ denotes the smallest integer greatest than x. In particular,

$$h_n^p = (n-1)(n-2)h_{n-1}^p + h_{n-1}^i$$

where h_n^i denotes the number of helices associated to the preinjective module (n, n + 1).

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Corollary				
For $n \ge 3$ fixed	d, let Γ be the Brauer configuration Γ =	$= (\Gamma_0, \Gamma_1, \mathcal{O}, \mu)$ such that:		
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For $n \ge 3$ fixed, let Γ be the Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mathcal{O}, \mu)$ such that: $ \begin{array}{l} \bullet \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\$					
For $n \ge 3$ fixed, let Γ be the Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mathcal{O}, \mu)$ such that: \mathbf{O} $\Gamma_0 = \{x_1, x_2\},$ $\Gamma_1 = \{V_k = x_1^{(2k+2)!} x_2^{\binom{k}{2}(2k+2)!}\}_{1 \le k \le n}.$ (2) (3) The orientation \mathcal{O} is defined in such a way that for $n \ge 1$ At vertex x_1 : $V_1^{(4)} \le V_2^{(6)} \le V_3^{(6)} \le \cdots \le V_n^{\binom{(2n+2)!}{2}},$ $At vertex x_2: V_1^{(12)} \le V_2^{(720)} \le V_3^{(60480)} \le \cdots \le V_n^{\binom{(2n+2)!}{2}}, \mu(\alpha) = 1, \text{ for any vertex } \alpha \in \Gamma_0. Then there exists a bijective correspondence between indecomposable projective \Lambda_{\Gamma}-modules and indecomposable preprojective Kronecker modules of the form (2k + 3, 2k + 2), 1 \le k \le n.$					
For $n \ge 3$ fixed, let Γ be the Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mathcal{O}, \mu)$ such that: $T_0 = \{x_1, x_2\},$ $\Gamma_0 = \{x_1, x_2\},$ $\Gamma_1 = \{V_k = x_1^{(2k+2)!} x_2^{\binom{k}{2}(\frac{2k+2)!}} \}_{1 \le k \le n}.$ (2) The orientation \mathcal{O} is defined in such a way that for $n \ge 1$ At vertex x_1 : $V_1^{(4)} \le V_2^{(6)} \le V_3^{(8)} \le \dots \le V_n^{\binom{(2n+2)!}{2}},$ $At vertex x_2: V_1^{(12)} \le V_2^{(720)} \le V_3^{(60480)} \le \dots \le V_n^{\binom{(2n+2)!}{2}}, \mu(\alpha) = 1, \text{ for any vertex } \alpha \in \Gamma_0. Then there exists a bijective correspondence between indecomposable projective \Lambda_{\Gamma}-modules and indecomposable preprojective Kronecker modules of the form (2k + 3, 2k + 2), 1 \le k \le n.$					
For $n \ge 3$ fixed, let Γ be the Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mathcal{O}, \mu)$ such that: $\Gamma_0 = \{x_1, x_2\},$ $\Gamma_1 = \{V_k = x_1^{(2k+2)!} x_2^{\binom{k}{2}(2k+2)!}\}_{1 \le k \le n}.$ (2) (2) The orientation \mathcal{O} is defined in such a way that for $n \ge 1$ At vertex x_1 ; $V_1^{(4)} \le V_2^{(6)} \le V_3^{(8)} \le \dots \le V_n^{((2n+2)!)},$ At vertex x_2 ; $V_1^{(12)} \le V_2^{(720)} \le V_3^{(60480)} \le \dots \le V_n^{\binom{((n)(2n+2)!)}{2}},$ (3) $\mu(\alpha) = 1, \text{ for any vertex } \alpha \in \Gamma_0.$ Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \le k \le n.$	Corollar	у			
$ \begin{aligned} \mathbf{O} \\ & \Gamma_0 = \{x_1, x_2\}, \\ & \Gamma_1 = \{V_k = x_1^{(2k+2)!}, x_2^{(k)(2k+2)!}\}_{1 \le k \le n}. \end{aligned} $ $ \begin{aligned} \mathbf{O} \text{ is defined in such a way that for } n \ge 1 \end{aligned} $ $ At \text{ vertex } x_1; \ V_1^{(4)} \le V_2^{(6)} \le V_3^{(8)} \le \cdots \le V_n^{((2n+2)!)}, \\ & At \text{ vertex } x_2; \ V_1^{(12)} \le V_2^{(720)} \le V_3^{(60480)} \le \cdots \le V_n^{((m)(2n+2)!)}, \\ & \mu(\alpha) = 1, \text{for any vertex } \alpha \in \Gamma_0. \end{aligned} $ $ \end{aligned} $ $ \textbf{Then there exists a bijective correspondence between indecomposable projective A_{\Gamma}-modules and indecomposable preprojective Kronecker modules of the form (2k + 3, 2k + 2), 1 \le k \le n. \end{aligned} $	For $n \ge 3$ fix	red, let Γ be the Brauer configuration Γ =	= $(\Gamma_0, \Gamma_1, \mathcal{O}, \mu)$ such that:		
$\Gamma_{0} = \{x_{1}, x_{2}\},$ $\Gamma_{1} = \{V_{k} = x_{1}^{(2k+2)!} x_{2}^{\binom{(k)(2k+2)!}{2}}\}_{1 \le k \le n}.$ (2) The orientation \mathcal{O} is defined in such a way that for $n \ge 1$ At vertex x_{1} : $V_{1}^{(4)} \le V_{2}^{(6)!} \le V_{3}^{(8)!} \le \cdots \le V_{n}^{\binom{(2n+2)!}{2}},$ At vertex x_{2} : $V_{1}^{(12)} \le V_{2}^{(720)} \le V_{3}^{(60480)} \le \cdots \le V_{n}^{\binom{(m)(2n+2)!}{2}},$ (3) $\mu(\alpha) = 1, \text{ for any vertex } \alpha \in \Gamma_{0}.$ (3) Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable projective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \le k \le n.$	0				
(2) $\Gamma_{1} = \{V_{k} = x_{1}^{(2k+2)!} x_{2}^{\binom{k}{2}(2k+2)!}\}_{1 \le k \le n}.$ (2) The orientation \mathcal{O} is defined in such a way that for $n \ge 1$ At vertex x_{1} ; $V_{1}^{(4)} \le V_{2}^{(6)} \le V_{3}^{(8)} \le \dots \le V_{n}^{\binom{(2n+2)!}{2}},$ At vertex x_{2} ; $V_{1}^{(12)} \le V_{2}^{(720)} \le V_{3}^{(60480)} \le \dots \le V_{n}^{\binom{(2n+2)!}{2}},$ $\mu(\alpha) = 1, \text{ for any vertex } \alpha \in \Gamma_{0}.$ (3) Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \le k \le n.$		$\Gamma_0 = \{x_1, x_2\}$,		
$I_{1} = \{V_{k} = x_{1}^{1} \cdots x_{2}^{n} \\ I_{1} \leq k \leq n^{*} \}$ The orientation O is defined in such a way that for $n \geq 1$ At vertex x_{1} : $V_{1}^{(4)} \leq V_{2}^{(6)} \leq V_{3}^{(8)} \leq \cdots \leq V_{n}^{((2n+2))},$ At vertex x_{2} : $V_{1}^{(12)} \leq V_{2}^{(720)} \leq V_{3}^{(60480)} \leq \cdots \leq V_{n}^{(\frac{(m(2n+2)!}{2}))},$ $\mu(\alpha) = 1, \text{ for any vertex } \alpha \in \Gamma_{0}.$ Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \leq k \leq n.$			$(2k+2)! (\frac{(k)(2k+2)!}{2})$		(2)
 (2) The orientation O is defined in such a way that for n ≥ 1 At vertex x₁; V₁⁽⁴⁾ ≤ V₂⁽⁶⁾ ≤ V₃⁽⁸⁾ ≤ ··· ≤ V_n^{((2n+2)!)}, At vertex x₂; V₁⁽¹²⁾ ≤ V₂⁽⁷²⁰⁾ ≤ V₃⁽⁶⁰⁴⁸⁰⁾ ≤ ··· ≤ V_n^{(((n)(2n+2)!)}), (3) µ(α) = 1, for any vertex α ∈ Γ₀. (3) the multiplicity function µ is such that µ(j) = 1, for any j ∈ Γ₀. Then there exists a bijective correspondence between indecomposable projective Λ_Γ-modules and indecomposable preprojective Kronecker modules of the form (2k + 3, 2k + 2), 1 ≤ k ≤ n. 		$\Gamma_1 = \{V_k = x\}$	$1 \qquad x_2 \qquad y_{1 \le k \le n}$		
At vertex x_1 : $V_1^{(41)} \leq V_2^{(61)} \leq V_3^{(81)} \leq \cdots \leq V_n^{((2n+2)!)}$, At vertex x_2 : $V_1^{(12)} \leq V_2^{(720)} \leq V_3^{(60480)} \leq \cdots \leq V_n^{(\frac{(m)(2n+2)!}{2})}$, (3) $\mu(\alpha) = 1$, for any vertex $\alpha \in \Gamma_0$. (3) the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$. Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \leq k \leq n$.	2 The c				
At vertex x_1 : $V_1^{(4)} \leq V_2^{(61)} \leq V_3^{(81)} \leq \cdots \leq V_n^{((2n+2)!)}$, At vertex x_2 : $V_1^{(12)} \leq V_2^{(720)} \leq V_3^{(60480)} \leq \cdots \leq V_n^{((\frac{(n)(2n+2)!}{2}))}$, (3) $\mu(\alpha) = 1$, for any vertex $\alpha \in \Gamma_0$. (3) the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$. Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \leq k \leq n$.					
At vertex x_1 : $V_1^{(4)} \le V_2^{(6)} \le V_3^{(8)} \le \cdots \le V_n^{((2n+2)!)}$, At vertex x_2 : $V_1^{(12)} \le V_2^{(720)} \le V_3^{(60480)} \le \cdots \le V_n^{((\frac{(n)(2n+2)!}{2}))}$, (3) $\mu(\alpha) = 1$, for any vertex $\alpha \in \Gamma_0$. (3) the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$. Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \le k \le n$.					
At vertex v_2 ; $V_1^{(12)} \leq V_2^{(720)} \leq V_3^{(60480)} \leq \cdots \leq V_n^{(\left(\frac{(n)(2n+2)!}{2}\right))}$. (3) $\mu(\alpha) = 1$, for any vertex $\alpha \in \Gamma_0$. (3) the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$. Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \leq k \leq n$.					
At vertex x_0 : $V_1 \to \leq V_2 \to \leq V_3 \to \leq \cdots \leq V_n$ $\mu(\alpha) = 1$, for any vertex $\alpha \in \Gamma_0$. (3) the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$. Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \leq k \leq n$.					
$\mu(\alpha) = 1$, for any vertex $\alpha \in I_0$. (3) the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$. Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \le k \le n$.					
3 the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$. Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \le k \le n$.					
Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2)$, $1 \le k \le n$.	(3) the n				
	Then there expreprojective	xists a bijective correspondence between Kronecker modules of the form (2k + 3,	indecomposable projective Λ_{Γ} -m 2k + 2), 1 < k < n.	odules and indecomp	osable
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Corollary					
For $n \ge 3$ fixed,	let Γ be the Brauer configuration	ion $\Gamma = ($	$(\Gamma_0,\Gamma_1,\mathcal{O},\mu)$ such that:		
	$\Gamma_0 = \{x\}$	1, x ₂ },			
	$\Gamma_1 = \{V$	$V_k = x_1^{(2k)}$	$+2)! x_{2}^{\left(\frac{(k)(2k+2)!}{2}\right)} }_{x_{2}} \}_{1 \le k \le n}.$		(2)
2 The orie	ntation ${\mathcal O}$ is defined in such a	way that	for $n \ge 1$		

At vertex
$$x_1$$
: $V_1^{(4!)} \le V_2^{(6!)} \le V_3^{(8!)} \le \dots \le V_n^{((2n+2)!)}$,
At vertex x_2 : $V_1^{(12)} \le V_2^{(720)} \le V_3^{(60480)} \le \dots \le V_n^{((\frac{n}{2}(2n+2)!))}$,
 $\mu(\alpha) = 1$, for any vertex $\alpha \in \Gamma_0$.
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the multiplicity function μ is such that $\mu(j)=1$, for any $j\in \Gamma_0$.

Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \le k \le n$.

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Corollary	,			

For $n\geq 3$ fixed, let Γ be the Brauer configuration $\Gamma=(\Gamma_0,\Gamma_1,\mathcal{O},\mu)$ such that:

$$\Gamma_{0} = \{x_{1}, x_{2}\},$$

$$\Gamma_{1} = \{V_{k} = x_{1}^{(2k+2)!} x_{2}^{(\frac{(k)(2k+2)!}{2})}\}_{1 \le k \le n}.$$
(2)

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At vertex
$$x_1$$
; $V_1^{(4!)} \le V_2^{(6!)} \le V_3^{(8!)} \le \dots \le V_n^{((2n+2)!)}$,
At vertex x_2 ; $V_1^{(12)} \le V_2^{(720)} \le V_3^{(60480)} \le \dots \le V_n^{((\frac{(n)(2n+2)!}{2}))}$, (3)
 $\mu(\alpha) = 1$, for any vertex $\alpha \in \Gamma_0$.

3 the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$. Then there exists a bijective correspondence between indecomposable projective Λ_{Γ} -modules and indecomposable preprojective Kronecker modules of the form $(2k + 3, 2k + 2), 1 \le k \le n$.

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Proof.

The specialization $x_1 = 1$, $x_2 = 2$ makes of each polygon V_k , $k \ge 1$ a unique partition λ of the number

$$h_{2k+2}^p = (2k+2)!\lceil k+1\rceil$$

into parts $\{1,2\}$ where $occ(x_i, V_k)$ coincides with the number of times that the part x_i occurs in the corresponding partition since h_{2k+2}^p gives the number of helices associated in a unique form to the indecomposable preprojective Kronecker module (2k + 3, 2k + 2).

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Helices and Exception	al Sequences			

Helices and Exceptional Sequences

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Helices and Exception	al Sequences			

P.F. Fernandez et al proved recently the following result which establishes a relationship between some helices and some exceptional sequences:

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Helices and Exceptio	nal Sequences			
		Auslander- Reiten guive	er of \mathbb{A}_3 .	



 $(X_{0,3}, X_{2,3}, X_{1,2})$

For each integers $0 \le i < j \le n$ we write X_{ij} the indecomposable whose representations is given by

$$1 \quad i \quad i+1 \quad j \quad j+1 \quad n \\ (0 \leftarrow \dots \leftarrow 0 \leftarrow k \leftarrow \dots \leftarrow k \leftarrow 0 \quad \leftarrow \dots \leftarrow 0)$$

Figure: Helices associated to some exceptional sequences. For notation see T. Araya, *Exceptional sequences over path algebras of type* \mathbb{A}_n and *non-crossing spanning trees*, Algebr. Represent. Theory, **16** (1), 239-250,

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Helices and Exceptiona	I Sequences			

Theorem

If (n + 1, n) denotes an indecomposable preprojective Kronecker module then helices of the form;

 $a_{1,1}, b_{1,1}, b_{n+1,1}, a_{n+1,n}, a_{n,n}, b_{n,n}, b_{n-1,n}, a_{n-1,n-2}, \dots, a_{3,2}, a_{2,2}, b_{2,2}$ when n is even.

 $a_{1,1}, b_{1,1}, b_{n+1,1}, a_{n+1,n}, a_{n,n}, b_{n,n}, b_{n-1,n}, \dots, b_{3,3}, b_{2,3}, a_{2,1}$ when n is odd.

correspond to complete exceptional sequences of type \mathbb{A}_n .

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The Four Subspace Prob	lem (FSP)			

The Four Subspace Problem

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The Four Subspace P	roblem (FSP)			

The four subspace problem consists of classifying all indecomposable quadruples (indecomposable representations of four incomparable points as a poset) up to isomorphism.

Zavadskij and Medina gave an elementary solution of this problem (2004).

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The Four Subspace Problem (FSP)

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The Four Subspace Problem (FSP)

The following result establishes a bijection between preprojective representations of type IV and indecomposable projective modules over some Brauer configuration algebras.

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The Four	Subspace Pr	oblem (FSP)			
	hoorom				
Eo	r n > 2 five	d let E be the Brower configuration E	$-(\Gamma_0, \Gamma_1, (2, \mu))$ such that:		
10			$f_{1} = (f_{0}, f_{1}, O, \mu)$ such that.		
	•				
		$\Gamma_0 = \{1, 2,$	$3, n, n+1$ }		(4)
		$\Gamma_1 = \{V_k\}$	$1 \le k \le n$, $V_i \ne V_j$ if $i \ne j$.		()
	3 the m				
Th	en there exi	sts a bijective correspondence between i	indecomposable projective Λ_{Γ_n} -n	nodules and indecomp	osable
pre	projective re	\geq	2 of the tetrad.		

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The Four Subspace Pr	roblem (FSP)			

Theorem

For $n \ge 2$ fixed, let Γ_n be the Brauer configuration $\Gamma_n = (\Gamma_0, \Gamma_1, \mathcal{O}, \mu)$ such that:

2 The orientation O is defined in such a way that

 $\operatorname{occ}(1, V_1) = 1$, $\operatorname{occ}(n + 1, V_n) = n + 1$, and for $2 \le i \le n$ at vertex *i*, $V_{i-1}^{(i+1,<)} < V_i^{(i^2,<)}$, where $V_y^{(x,<)}$ means that the polygon V_y occurs x times in the successor sequence of the corresponding vertex,

the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$.

Then there exists a bijective correspondence between indecomposable projective Λ_{Γ_n} -modules and indecomposable preprojective representations of type IV and order $n \ge 2$ of the tetrad.

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The Four Subspace Prob	olem (FSP)			

Theorem

For $n \ge 2$ fixed, let Γ_n be the Brauer configuration $\Gamma_n = (\Gamma_0, \Gamma_1, \mathcal{O}, \mu)$ such that:

2 The orientation O is defined in such a way that

 $\operatorname{occ}(1, V_1) = 1$, $\operatorname{occ}(n + 1, V_n) = n + 1$, and for $2 \le i \le n$ at vertex *i*, $V_{i-1}^{(i+1,<)} < V_i^{(i^2,<)}$, where $V_V^{(x,<)}$ means that the polygon V_Y occurs x times in the successor sequence of the corresponding vertex,

3 the multiplicity function μ is such that $\mu(j) = 1$, for any $j \in \Gamma_0$. Then there exists a bijective correspondence between indecomposable projective Λ_{Γ_n} -modules and indecomposable preprojective representations of type IV and order n > 2 of the tetrad.

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The Four Subspace P	roblem (FSP)			

Firstly, we note that the Brauer configuration (4) allows to see each polygon V_n as a partition of the number h_n into two parts of the form $\{n, n + 1\}$ where n occurs $(n)^2$ times and n + 1 occurs n + 1 times. Assuming the classical notation for partitions [2] each number h_n can be expressed as follows:

$$h_n = (n)^{(n^2)} (n+1)^{(n+1)}, \quad n \ge 1.$$
 (25)

we let P_n denote such a partition. The following is the quiver Q_{Γ_n} associated to such Brauer configuration. In this case, we use the symbol [x; y] to denote that the vertex x occurs y times at the corresponding polygon.



For instance:

$$\begin{aligned} 5 &= (1) + (2+2) \\ 17 &= (2+3) + (2+3) + (2+2+3) \\ 43 &= (3+3+4) + (3+3+4) + (3+3+4) + (3+3+3+4) \\ 89 &= (4+4+4+5) + (4+4+4+5) + (4+4+4+5) + (4+4+4+5) + (4+4+4+5) \end{aligned}$$

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Matrix Problems Associated to Some Brauer Configuration AlgebrasMaurice Auslander Distinguished LecturesFalmouth-USA

Aims and Scope	Brauer Configuration Algebras	Some Matrix Problems	Helices 0000	References

- Categorification of some integer sequences, A. M. Cañadas, H. Giraldo, P.F.F. Espinosa, FMJS, 92, 2014, no. 2, 125-139.
- A partition formula for Fibonacci numbers, P. Fahr, C. M. Ringel, Journal of integer sequences, 11, 2008, no. 08.14.
- Brauer Configuration Algebras: A Generalization of Brauer Graph Algebras, E.L. Green, S. Schroll, Bull. Sci. Math., 141, 2017, 539-572, 2017.
- A052558, A100705, OEIS (On-Line Encyclopedia of Integer Sequences).
- The four subspace problem; An elementary solution, A.G. Zavadskij, G.Medina, Linear Algebra App, 392, 11-23, 2004.

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