Annihilation of Cohomology over Curve Singularities

Maurice Auslander International Conference

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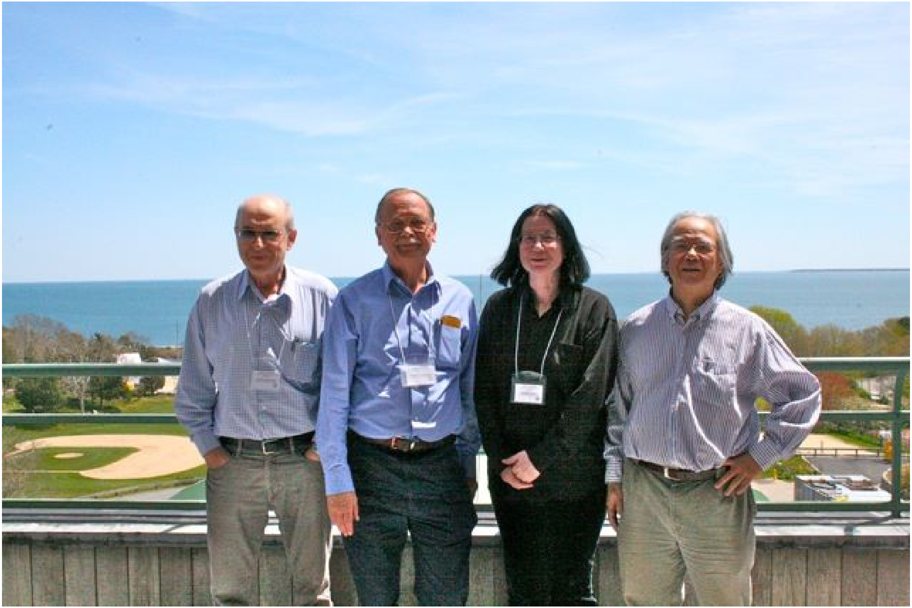

Definition and Motivation

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Definition

The cohomology annihilator ideal is the union

$$\operatorname{ca}(R) = \bigcup_{n \geq 1} \operatorname{ca}^n(R).$$

- For any $n \geq 1$ and $M, N \in \operatorname{mod} R$, we have

$$\operatorname{Ext}_R^{n+1}(M, N) \cong \operatorname{Ext}_R^n(\Omega M, N)$$

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- Therefore, we have an increasing chain

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- As R is Noetherian, we have

$$\operatorname{ca}(R) = \operatorname{ca}^s(R)$$

for $s \gg 0$.

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- Therefore,

$$\operatorname{ca}(R) = R.$$

- So, this ideal is only interesting in the case of infinite global dimension.

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Theorem (Iyengar-Takahashi)

Let R be an equicharacteristic excellent local ring or a localization of a finitely generated algebra over a field - of Krull dimension d. Then, the vanishing locus of $\mathrm{ca}(R)$ is equal to the singular locus of R.

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An Example

Let k be an algebraically closed field of characteristic zero, $f = x^3 - y^5$ and

$$R = \frac{k[\![x, y]\!]}{(f)}.$$

Then, the Jacobian and the cohomology annihilator ideals are

$$\begin{aligned} \mathrm{Jac}(R) &= (x^2, y^4) \qquad\qquad (= (\partial_x f, \partial_y f)) \\ \mathrm{ca}(R) &= (x^2, xy, y^3) \end{aligned}$$

(Details on computations later.)

Ragnar's Observation

$\mathrm{Jac}(R) = (x^2, y^4)$ $\qquad\qquad$ $\mathrm{ca}(R) = (x^2, xy, y^3)$

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\begin{array}{ccc}
 & 1 & \\
x & & y \\
xy & & y^2 \\
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That is, the vector space dimensions have the relation

$$\dim_k(R/\,\mathrm{Jac}(R)) = 2 \times \dim_k(R/\,\mathrm{ca}(R)).$$

Ragnar's Observation

- The same phenomenon can also be seen in the following examples:

$$k[\![x,y]\!]/(x^2 - y^n) \qquad (n \text{ odd}),$$
$$k[\![x,y]\!]/(x^3 - y^4),$$
$$k[\![x,y,z]\!]/(x^3 + y^3 + z^3 - \lambda xyz) \qquad (\lambda^3 \neq 27).$$

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\end{aligned}
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- Why? This was the question Ragnar asked me.

Figure: (Up to isomorphism of pictures) From left to right: Ragnar Buchweitz, Louis-Philippe Thibault, Vincent Gelinas, Ben Briggs and me.

The Singularity Category

From now on, we will assume that R is a commutative Gorenstein ring of Krull dimension d.

- $\underline{\mathrm{MCM}}(R)$: stable category of maximal Cohen-Macaulay modules.
- $D_{\mathrm{sg}}(R)$: the singularity category - the bounded derived category modulo perfect complexes.

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Theorem (Buchweitz)

$\underline{\mathrm{MCM}}(R) \cong D_{\mathrm{sg}}(R)$ *as triangulated categories.*

- For $M \in \operatorname{mod} R$, we denote by M^{st} the maximal Cohen-Macaulay approximation of M.

Stable Ext

- The usual Ext groups:

$$\mathrm{Ext}^n_R(M, N) = \mathrm{Hom}_{D^b(R)}(M, N[n]).$$

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- For any $M, N \in \mathrm{mod}(R)$, one has

$$\begin{aligned}
\underline{\mathrm{Ext}}_R^n(M, N) &= \mathrm{Ext}_R^n(M, N) \quad \text{for any } n > d, \\
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Back to the Problem

Lemma (?)

Let R be a commutative Gorenstein ring. Then,

$$\mathrm{ca}(R) = \bigcap_{M \in \mathrm{MCM}(R)} \underline{\mathrm{ann}}_R(M)$$

where $\underline{\mathrm{ann}}_R(M) = \mathrm{ann}\,\underline{\mathrm{End}}_R(M)$ *is the stable annihilator of* M.

Proof.

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So,

$$\begin{aligned}
\mathrm{ca}(R) &= \bigcap_{M,N \in \mathrm{MCM}(R)} \mathrm{ann}_R\, \underline{\mathrm{Hom}}_R(\Omega^s M, N) \\
&= \bigcap_{M,N \in \mathrm{MCM}(R)} \mathrm{ann}_R\, \underline{\mathrm{Hom}}_R(M, N) \\
&= \bigcap_{M \in \mathrm{MCM}(R)} \mathrm{ann}_R\, \underline{\mathrm{End}}_R(M)
\end{aligned}$$

□

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- This description is useful in computations. Indeed, one can compute cohomology annihilator over a hypersurface ring using matrix factorizations.
- In terms of matrix factorizations, r is in the cohomology annihilator if and only if multiplication with r is null-homotopic for every matrix factorization.

Remarks - II

- With this description, it is easy to show that if r stably annihilates M, then it annihilates every object in the smallest subcategory of $\underline{\mathrm{MCM}}(R)$ containing M and closed under finite direct sums, direct summands, syzygies, cosyzygies and duals. We will revisit this later.

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- So,

$$\underline{\mathrm{ann}}_R(M) = \mathrm{ann}_R \bigoplus_{n \in \mathbb{Z}} \underline{\mathrm{Hom}}(\Omega^n M, M)$$

The Coin Problem

Question. We know that we can write any integer as an integer linear combination of 3 and 5(Euclidean algorithm). Which numbers can we write using only non-negative linear combinations?

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any number after 7 can be obtained this way.

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any number after 7 can be obtained this way. On the other hand, notice that we have

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Conductor

Let R be a commutative Noetherian ring and let $\bar{R}$ be its normalization: the integral closure of R inside its total quotient ring. Then, the conductor ideal $\mathrm{co}(R)$ is

$$\mathrm{co}(R) = \{r \in \bar{R} \ : \ r\bar{R} \subseteq R\}$$

$\mathrm{co}(R)$ is the largest subset of $\bar{R}$ which is both an ideal of R and $\bar{R}$.

Theorem (Wang)

Let R be a one-dimensional reduced complete Noetherian local ring. Then, $\mathrm{co}(R) \subseteq \mathrm{ann}_R \mathrm{Ext}^2(M, N)$ for any $M, N \in \mathrm{mod}\, R$.

In other words, for a one-dimensional reduced complete Noetherian local ring R ; $\mathrm{co}(R) \subseteq \mathrm{ca}(R)$.

Conductor

Lemma

Let R be any Japanese ring. Then,

$$\underline{\mathrm{End}}_R(\bar{R}) \cong \frac{\bar{R}}{\mathrm{co}R}$$

as R-modules via the isomorphism $f \mapsto \overline{f(1)}$.

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- If R is a one or two dimensional Gorenstein ring, then $\bar{R}$ is maximal Cohen-Macaulay over R.
- Hence,

$$\mathrm{ca}(R) \subseteq \underline{\mathrm{ann}}_R\, \bar{R} = \mathrm{ann}_R\, \frac{\bar{R}}{\mathrm{co}R} = \mathrm{co}(R)$$

Theorem (E.)

Let R be a one-dimensional complete local Gorenstein ring. Then,

$$\mathsf{ca}(R) \subseteq \mathrm{co}(R).$$

If R is also reduced then there is equality.

Back to Ragnar's Observation

Ragnar's observation is now explained via the famous Milnor-Jung formula for algebraic curves:

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Theorem (Milnor-Jung Formula)

Let C be a reduced irreducible curve with an isolated singular point. Let R be the coordinate ring of C - localized at this singular point. Then,

$$\dim_k \frac{R}{\mathrm{Jac}(R)} = 2\dim_k \frac{R}{\mathrm{co}(R)} - r + 1$$

where r is the number of branches at the singular point.

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Example

Let G be a finite subgroup of $\mathrm{SL}(2,\mathbb{C})$, $S = k[\![x,y]\!]$ and $R = S^G$ be the invariant ring. Then,

$$\mathrm{ca}(R) = \underline{\mathrm{ann}}_R(S) = \underline{\mathrm{ann}}_R(S * G)$$

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- In general?

Theorem (E.)

Let R be a commutative Gorenstein ring and Λ be a finite R-algebra of finite global dimension. Suppose that R is a direct summand in Λ. Then,

$$\operatorname{ann}_R \operatorname{End}_{D_{sg}(R)}(\Lambda)^{\operatorname{gldim}\Lambda+1} \subseteq \operatorname{ca}(R) \subseteq \operatorname{ann}_R \operatorname{End}_{D_{sg}(R)}(\Lambda)$$

If, in addition, $\Lambda \in \operatorname{MCM}(R)$ then

$$\underline{\operatorname{ann}}_R(\Lambda)^{\operatorname{gldim}\Lambda+1} \subseteq \operatorname{ca}(R) \subseteq \underline{\operatorname{ann}}_R(\Lambda)$$

In particular, up to radicals, in order to annihilate the singularity category, it is enough to stably annihilate a noncommutative resolution.

Thank you!