String and Band Complexes over Certain Algebra of Dihedral Type

José A. Vélez-Marulanda

Valdosta State University

Joint work with

Hernán Giraldo

Universidad de Antioquia

Maurice Auslander Distinguished Lectures and International Conference,
Woods Hole, MA, May 2, 2016

Set Up

In this talk:

- $\mathbb{k}$ is an algebraically closed field of arbitrary characteristic.
- The Λ's always denote finite-dimensional $\mathbb{k}$-algebras.
- Unless explicitly stated otherwise, all our modules are modules from the right.
- We denote by $\text{mod}\,\Lambda$ the abelian category of finitely generated right Λ-modules, and $\mathscr{P}_\Lambda$ denotes the full subcategory of $\text{mod}\,\Lambda$ whose objects are finitely generated projective Λ-modules.
- $\mathcal{K}^b(\mathscr{P}_\Lambda)$ denotes the triangulated category of perfect complexes over Λ and $\mathcal{D}^b(\text{mod}\,\Lambda)$ denotes the bounded derived category of $\text{mod}\,\Lambda$.

MOTIVATION & BACKGROUND

Let $V^\bullet$ be an object of $\mathcal{D}^-(\mathrm{mod}\,\Lambda)$ that has finitely many non-zero cohomology groups, all which have finite dimension over $\Bbbk$.

- In 2015, F. M. BLEHER and V-M proved that $V^\bullet$ has a well-defined versal deformation ring $R(\Lambda, V^\bullet)$, which is a complete local commutative Noetherian $\Bbbk$-algebra with residue field $\Bbbk$. Moreover, the isomorphism class of $R(\Lambda, V^\bullet)$ is preserved under derived equivalences.
- They also proved that versal deformation rings of modules are preserved under stable equivalences of Morita type (as introduced by M. BROUÉ in 1994) between self-injective $\Bbbk$-algebras.
- In 2016, in an ongoing research, V-M proved that versal deformation rings of Cohen-Macaulay modules are preserved under **singular equivalences of Morita type** between Gorenstein $\Bbbk$-algebras.
- These singular equivalences of Morita type where introduced in a preprint by X. W. CHEN and L. G. SUN during 2012 and then formally discussed in a published article by G. ZHOU and A. ZIMMERMANN in 2013.

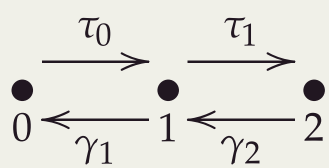
The ultimate goal is to use “nice" descriptions of such complexes $V^\bullet$ to explicitly describe $R(\Lambda, V^\bullet)$ for when Λ is e.g. a Gorenstein $\Bbbk$-algebra.

- In general, it is a difficult problem to describe the indecomposable objects in $\mathcal{D}^b(\text{mod}\,\Lambda)$.
- Assume that Λ is a **gentle algebra** as introduced by I. ASSEM and A. SKOWROŃSKI in 1987.
- In 2003, V. BEKKERT and H. A. MERKLEN provided a combinatorial description of the indecomposable objects in $\mathcal{D}^b(\text{mod}\,\Lambda)$. They used so-called string and band complexes, which are indecomposable objects in $\mathcal{K}^b(\mathscr{P}_\Lambda)$.
- They used the obtained results to prove that gentle algebras are derived tame as introduced by CH. GEISS & H. KRAUSE in 2002.
- Then in 2011, G. BOBIŃSKI used these string and band complexes to describe the almost split triangles in $\mathcal{K}^b(\mathscr{P}_\Lambda)$.
- He also showed the relation between the description provided by V. BEKKERT and H. A. MERKLEN with the Happel functor $\mathbf{F} : \mathcal{D}^b(\text{mod}\,\Lambda) \to \underline{\text{mod}}\,\hat{\Lambda}$, where $\underline{\text{mod}}\,\hat{\Lambda}$ denotes the stable module category of the repetitive algebra $\hat{\Lambda}$.

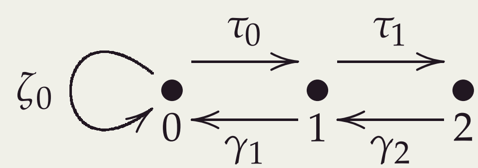
Question: How about self-injective non-gentle algebras?

Consider the following quivers.

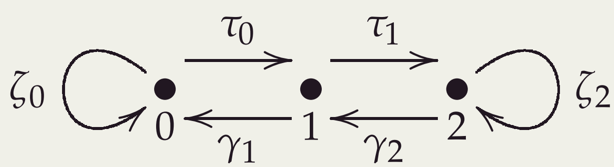
$3\mathscr{A} =$



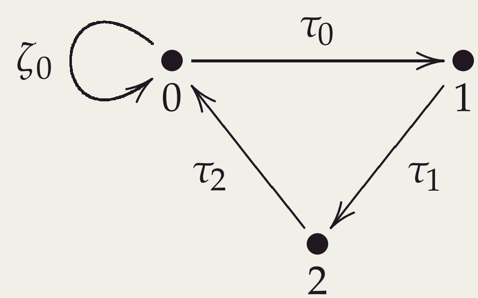
$3\mathscr{B} =$



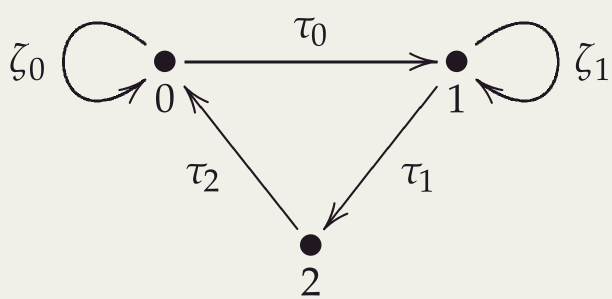
$3\mathscr{D} =$



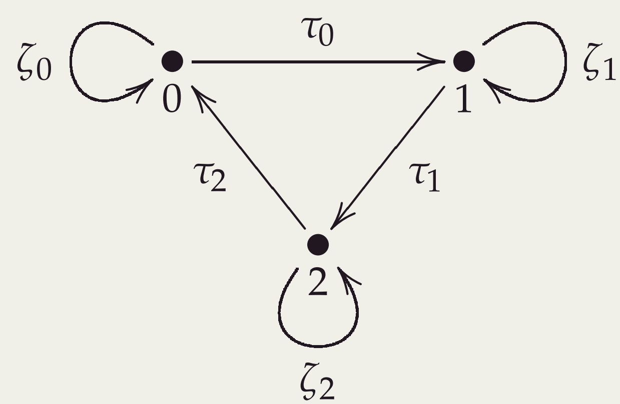
$3\mathscr{L} =$



$3\mathscr{Q} =$



$3\mathscr{R} =$



Let Λ be one of the following bounded path algebras.

$$D(3\mathscr{A})_2^{2,2} = \Bbbk[3\mathscr{A}]/\langle \tau_0\tau_1, \gamma_2\gamma_1, (\gamma_1\tau_0)^2 - (\tau_1\gamma_2)^2\rangle$$

$$D(3\mathscr{B})_2^{2,2,2} = \Bbbk[3\mathscr{B}]/\langle \gamma_1\zeta_0, \zeta_0\tau_0, \tau_0\tau_1, \gamma_2\gamma_1, (\gamma_1\tau_0)^2 - (\tau_1\gamma_2)^2, (\tau_0\gamma_1)^2 - \zeta_0^2\rangle$$

$$D(3\mathscr{D})_2^{1,2,2,2} = \Bbbk[3\mathscr{D}]/\langle \gamma_1\zeta_0, \zeta_0\tau_0, \tau_0\tau_1, \gamma_2\gamma, \tau_1\zeta_2, \zeta_2\gamma_2, \tau_0\gamma_1 - \zeta_0^2, (\gamma_2\tau_1)^2 - \zeta_2^2, \gamma_1\tau_0 - (\tau_1\gamma_2)^2\rangle$$

$$D(3\mathscr{L})^{2,2} = \Bbbk[3\mathscr{L}]/\langle \zeta_0\tau_0, \tau_2\zeta_0, (\tau_0\tau_1\tau_2)^2 - \zeta_0^2, (\tau_1\tau_2\tau_0)^2\tau_1\rangle$$

$$D(3\mathscr{Q})^{1,2,2} = \Bbbk[3\mathscr{Q}]/\langle \zeta_0\tau_0, \tau_2\zeta_0, \tau_0\zeta_1, \zeta_1\tau_1, \tau_0\tau_1\tau_2 - \zeta_0^2, \tau_1\tau_2\tau_0 - \zeta_1^2\rangle$$

$$D(3\mathscr{Q})^{2,2,2} = \Bbbk[3\mathscr{Q}]/\langle \zeta_0\tau_0, \tau_2\zeta_0, \tau_0\zeta_1, \zeta_1\tau_1, (\tau_0\tau_1\tau_2)^2 - \zeta_0^2, (\tau_1\tau_2\tau_0)^2 - \zeta_1^2\rangle$$

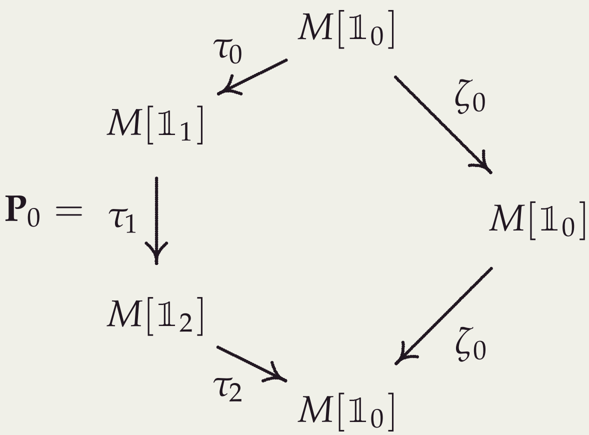
$$D(3\mathscr{R})^{1,2,2,2} = \Bbbk[3\mathscr{R}]/\langle \zeta_0\tau_0, \tau_0\zeta_1, \zeta_1\tau_1, \tau_1\zeta_2, \zeta_2\tau_2, \tau_2\zeta_0, \tau_0\tau_1\tau_2 - \zeta_0^2, \tau_1\tau_2\tau_0 - \zeta_1^2, \tau_2\tau_0\tau_1 - \zeta_2^2\rangle$$

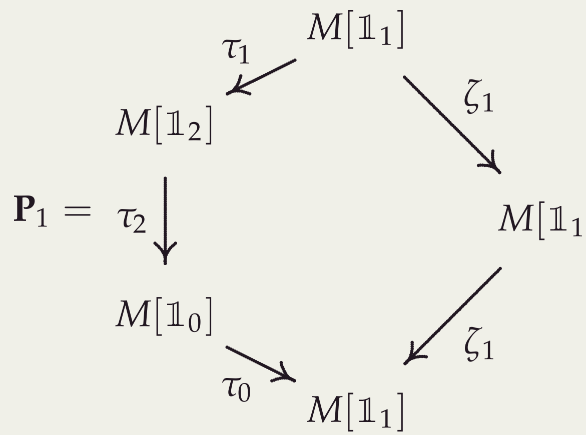
Theorem 1. (T. HOLM, 1999) *The algebras* $D(3\mathscr{B})_2^{2,2,2}$, $D(3\mathscr{D})_2^{1,2,2,2}$, $D(3\mathscr{Q})^{2,2,2}$ *and* $D(3\mathscr{R})^{1,2,2,2}$ *(resp.* $D(\mathscr{A})_2^{2,2}$, $D(\mathscr{L})^{2,2}$ *and* $D(3\mathscr{Q})^{1,2,2}$*) are derived equivalent.*

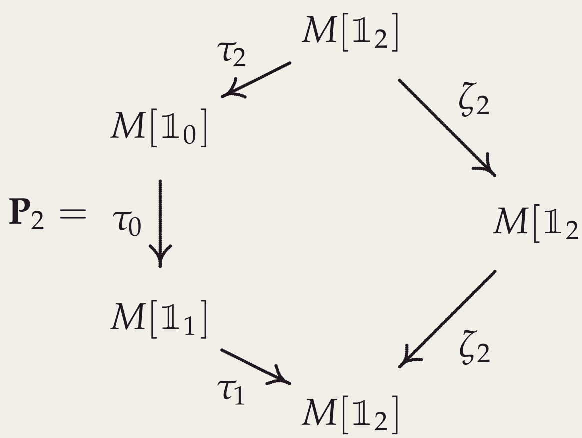
Thus, we can restrict ourselves to the algebras

$$\Lambda_0 = D(3\mathscr{R})^{1,2,2,2} \qquad \text{and} \qquad \Lambda_1 = D(3\mathscr{Q})^{1,2,2}.$$

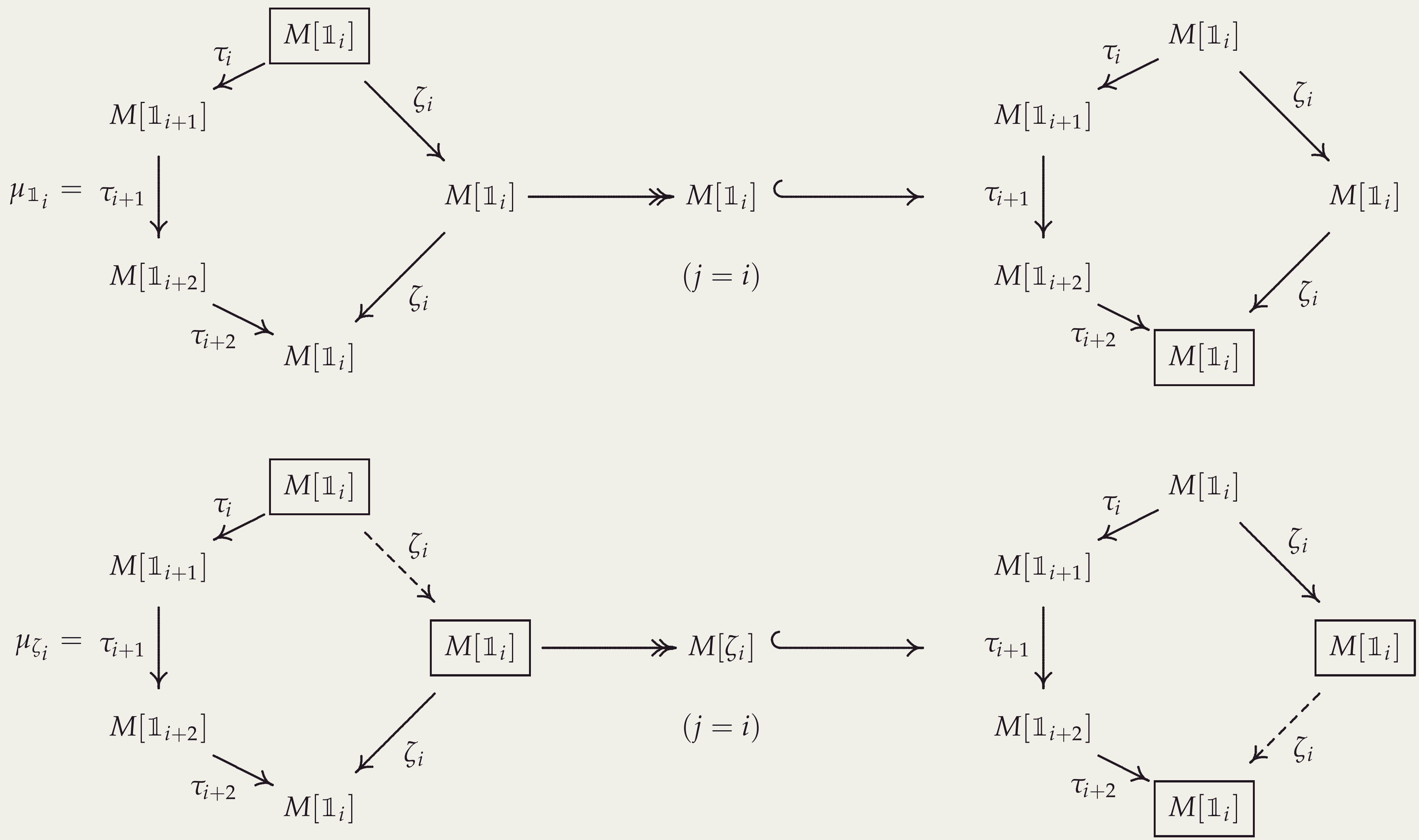
Remark 2. *The results obtained for* Λ_0 *can be adjusted for* Λ_1 *by letting* $\zeta_2 = \mathbb{1}_2$ *in the graph of* $3\mathscr{Q}$*, where* $\mathbb{1}_2$ *denotes the path of length zero that starts and ends at the vertex* 2*.*


$\mathbf{P}_0 =$
$M[\mathbb{1}_0]$
τ_0
ζ_0
$M[\mathbb{1}_1]$
τ_1
$M[\mathbb{1}_0]$
$M[\mathbb{1}_2]$
ζ_0
τ_2
$M[\mathbb{1}_0]$

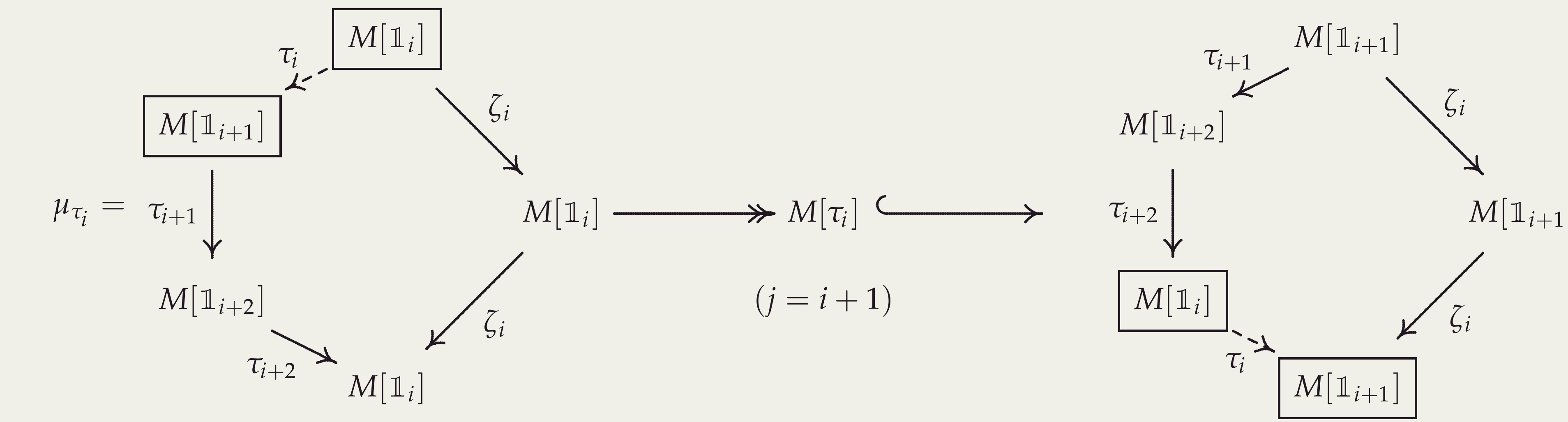

$\mathbf{P}_1 =$
$M[\mathbb{1}_1]$
τ_1
ζ_1
$M[\mathbb{1}_2]$
τ_2
$M[\mathbb{1}_1]$
$M[\mathbb{1}_0]$
ζ_1
τ_0
$M[\mathbb{1}_1]$

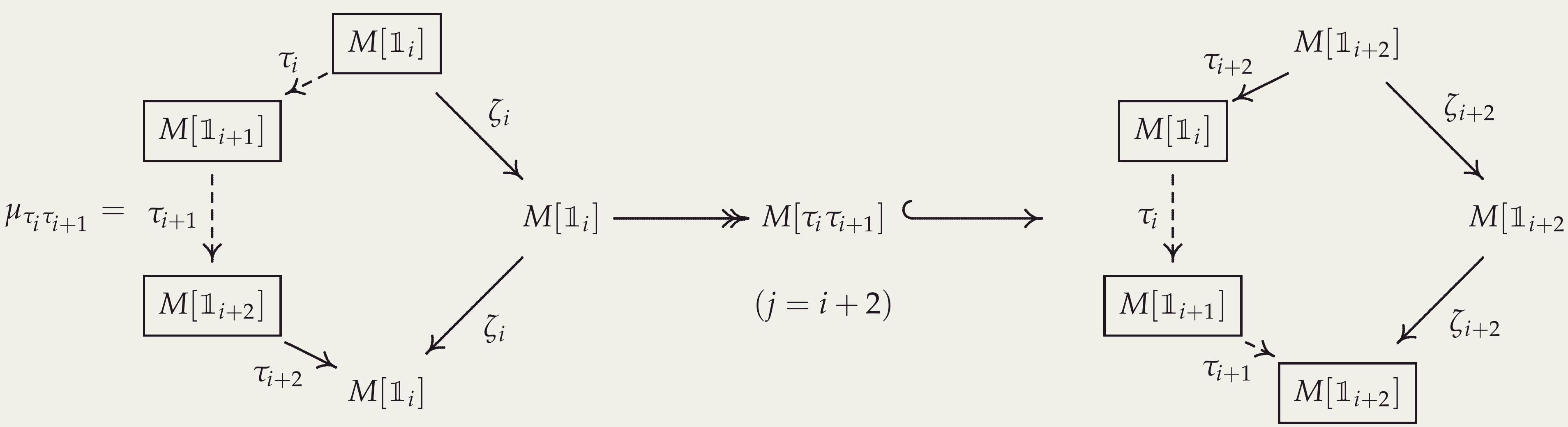

$\mathbf{P}_2 =$
$M[\mathbb{1}_2]$
τ_2
ζ_2
$M[\mathbb{1}_0]$
τ_0
$M[\mathbb{1}_2]$
$M[\mathbb{1}_1]$
ζ_2
τ_1
$M[\mathbb{1}_2]$

Canonical Morphisms Between Indecomposable Projective Λ_0-modules



Canonical Morphisms Between Indecomposable Projective Λ_0-modules





Observe that for all $i \in \{0,1,2\} \mod 3$, we have the following relations.

$$\mu_{\zeta_i}\mu_{\zeta_i} = \mu_{\mathbb{1}_i} \qquad \mu_{\zeta_i}\mu_{\mathbb{1}_i} = 0 \qquad \mu_{\mathbb{1}_i}\mu_{\zeta_i} = 0$$

$$\mu_{\zeta_{i+1}}\mu_{\tau_i} = 0 \qquad \mu_{\tau_i}\mu_{\zeta_i} = 0 \qquad \mu_{\mathbb{1}_{i+1}}\mu_{\tau_i} = 0$$

$$\mu_{\zeta_{i+2}}\mu_{\tau_i\tau_{i+1}} = 0 \qquad \mu_{\tau_i\tau_{i+1}}\mu_{\zeta_i} = 0 \qquad \mu_{\tau_{i+1}}\mu_{\tau_i} = 0$$

$$\mu_{\tau_i}\mu_{\mathbb{1}_i} = 0 \qquad \mu_{\tau_i\tau_{i+1}}\mu_{\mathbb{1}_i} = 0 \qquad \mu_{\mathbb{1}_{i+2}}\mu_{\tau_i\tau_{i+1}} = 0$$

$$\mu_{\tau_{i+1}\tau_{i+2}}\mu_{\tau_i} = \mu_{\mathbb{1}_i} \qquad \mu_{\tau_{i+2}}\mu_{\tau_i\tau_{i+1}} = \mu_{\mathbb{1}_i} \qquad \mu_{\tau_{i+2}\tau_i}\mu_{\tau_i\tau_{i+1}} = \mu_{\tau_i}.$$

Moreover, for all $i \in \{0,1,2\} \mod 3$, $\mu_{\mathbb{1}_i}$ is the morphism induced by the Λ_0-module isomorphism $\mathbf{P}_i/\mathrm{rad}(\mathbf{P}_i) \cong \mathrm{soc}(\mathbf{P}_i)$.

Let $i,j \in \{0,1,2\} \mod 3$. By (H. Krause, 1991), the non-trivial canonical morphisms generate $\mathrm{rad}_\Lambda(\mathbf{P}_i,\mathbf{P}_j)$ as a $\Bbbk$-vector space.

- We denote by $\mathbf{Pa}(\Lambda_0)$ the set of all paths in Λ_0, and by $\mathbf{Pa}_{>0}(\Lambda_0)$ all the paths whose length is greater than 0.
- If w is a path of positive length in Λ_0, we define a formal inverse w^{-1} of w and we let $\mathbf{s}(w^{-1}) = \mathbf{t}(w)$ and $\mathbf{t}(w^{-1}) = \mathbf{s}(w^{-1})$.
- By a generalized word for Λ_0 of positive length $n > 0$, we mean a sequence $w_1 \cdot w_2 \cdots w_n$ where each w_j is either a path of positive length, or the formal inverse of a path of positive length, and such that $\mathbf{s}(w_{j+1}) = \mathbf{t}(w_j)$ for $1 \leq j \leq n-1$, $\mathbf{s}(w) = \mathbf{s}(w_1)$ and $\mathbf{t}(w) = \mathbf{t}(w_n)$.
- If $w = w_1 \cdot w_2 \cdots w_n$ is a generalized word of length $n > 0$, we let $w^{-1} = w_n^{-1} \cdots w_2^{-1} \cdot w_1^{-1}$.
- If we have a word $w = w_1 \cdots w_n$ of positive length such that $\mathbf{s}(w) = \mathbf{t}(w)$ we say that w is a **closed generalized word** and for all $1 \leq j \leq n-1$, the j**-th word rotation** $w[j]$ of w is the word $w_{j+1} \cdots w_n \cdot w_1 \cdots w_j$.
- If v and w are two generalized words, we say that $w \sim_S v$ if and only if $w = v^{-1}$; and if v and w are further closed words, we say that $w \sim_R v$ if and only if either $w = v^{-1}$ or there exists $j \geq 1$ such that $w = v[j]$.
- If w is a closed generalized word of non-negative length, then for all integers $n \geq 1$, we denote by $w^{\cdot n}$ the n-fold generalized concatenation $w \cdot w \cdots w$ of w with itself, and we call $w^{\cdot n}$ a **generalized power** of w.

For all $i \in \{0,1,2\} \mod 3$, we assume that $\mathbb{1}_i$ is a generalized word of length zero and for all generalized words w for Λ_0 of positive length, we assume that $w \cdot \mathbb{1}_{\mathbf{t}(w)} \neq w \neq \mathbb{1}_{\mathbf{s}(w)} \cdot w$ as generalized words.

Let

$$J = \langle \zeta_0\tau_0, \tau_0\zeta_1, \zeta_1\tau_1, \tau_1\zeta_2, \zeta_2\tau_2, \tau_2\zeta_0, \zeta_0^2, \zeta_1^2, \zeta_2^2, \tau_0\tau_1\tau_2, \tau_1\tau_2\tau_0, \tau_2\tau_0\tau_1 \rangle,$$

and

$$J' = \langle \zeta_0^2, \zeta_1^2, \zeta_2^2, \tau_0\tau_1\tau_2, \tau_1\tau_2\tau_0, \tau_2\tau_0\tau_1 \rangle$$

be ideals of $\mathbb{k}[3\mathscr{R}]$.

We denote by $St(\Lambda_0)$ the set of all strings representatives for Λ_0.

We denote by $\overline{GSt(\Lambda_0)}$ the set of all generalized words of positive length $w = w_1 \cdot w_2 \cdots w_n$ that satisfies the following conditions. For all $1 \leq j \leq n-1$,

(i) if $w_j, w_{j+1} \in \mathbf{Pa}_{>0}(\Lambda_0)$, then $w_j w_{j+1} \in J - J'$;

(ii) if $w_j^{-1}, w_{j+1}^{-1} \in \mathbf{Pa}_{>0}(\Lambda_0)$, then $w_{j+1}^{-1} w_j^{-1} \in J - J'$;

(iii) if either $w_j, w_{j+1}^{-1} \in \mathbf{Pa}_{>0}(\Lambda_0)$ or $w_j^{-1}, w_{j+1} \in \mathbf{Pa}(\Lambda_0)$, then $w_j w_{j+1} \in St(\Lambda_0)$.

We denote by $GSt(\Lambda_0)$ a fixed set of representatives of the quotient of $\overline{GSt(\Lambda_0)}$ over the equivalence relation $\sim_S$, and the elements of $GSt(\Lambda_0)$ will be called **generalized strings** for Λ_0.

Generalized Strings (Bands) for Λ_0

- Note that for all $i \in \{0,1,2\} \mod 3$, we do not consider $\mathbb{1}_i$ as a generalized string.
- We define inductively a function η over the set of generalized strings for Λ_0 as follows. If $w = w_1 \cdot w_2 \cdots w_n$ is a generalized word of positive length for Λ_0 with $n \geq 1$, then for all $1 \leq j \leq n$, we let

$$\eta_w(j) = \begin{cases} 0, & \text{if } j = 0, \\ \eta_w(j-1)+1, & \text{if } w_j \in \mathbf{Pa}_{>0}(\Lambda_0), \\ \eta_w(j-1)-1, & \text{if } w_j^{-1} \in \mathbf{Pa}_{>0}(\Lambda_0). \end{cases}$$

- For all generalized strings $w = w_1 \cdot w_2 \cdots w_n$ for Λ_0 of positive length, we define

$$\deg w := \max\{\eta_w(j) | 0 \leq j \leq n\}.$$

- Let $\overline{GBa(\Lambda_0)}$ the set of all closed generalized strings $w = w_1 \cdots w_n$ for Λ_0 such that $w^{\cdot 2} \in \overline{GSt(\Lambda_0)}$, $\eta_w(0) = \eta_w(n)$, and such that w is not itself a power of one of its proper sub-words. We denote by $GBa(\Lambda_0)$ a fixed set of representatives of the quotient set of $\overline{Ba(\Lambda_0)}$ over the equivalence relation $\sim_R$, and we call the elements of $GBa(\Lambda_0)$ **generalized bands** for Λ_0.

Definition 3. Let $w = w_1 \cdots w_n$ be a generalized string for Λ_0 with $n \geq 1$. We define the complex $P[w]^\bullet$ in $\mathcal{K}^b(\mathscr{P}_{\Lambda_0})$ as follows. For all $l \in \mathbb{Z}$, we let

$$P[w]^l = \bigoplus_{j=0}^{n} \Delta(\eta_w(j), l)\mathbf{P}_{c_w(j)},$$

where Δ is the Kronecker delta, $c_w(0) = \mathbf{s}(w)$, and for all $1 \leq j \leq n$, $c_w(j) = \mathbf{t}(w_j)$. The differential maps are $\delta^i_{P[w]^\bullet} = (\delta^l_{jk,w})_{0\leq j,k\leq n}$, where for each $l \in \mathbb{Z}$,

$$\delta^l_{jk,w} = \begin{cases} \mu_{w_{j+1}}, & \text{if } w_{j+1} \in \mathbf{Pa}_{>0}(\Lambda_0),\ \eta_w(j) = l \text{ and } k = j+1, \\ \mu_{w_j^{-1}}, & \text{if } w_j^{-1} \in \mathbf{Pa}_{>0}(\Lambda_0),\ \eta_w(j) = l \text{ and } k = j-1, \\ 0, & \text{otherwise.} \end{cases}$$

We call $P[w]^\bullet$ the **string complex** corresponding to the generalized string w.

We denote by $\operatorname{ind}\Bbbk[x]$ the set of all indecomposable polynomials with coefficients over $\Bbbk$, which are not of the form x^d for some $d \geq 1$.

Definition 4. Let $w = w_1 \cdot w_2 \cdots w_n$ be a generalized band for Λ_0 and let $g \in \operatorname{ind}\Bbbk[x]$. We define $P[w,g]^\bullet$ in $\mathcal{K}^b(\mathscr{P}_{\Lambda_0})$ as follows. For all $l \in \mathbb{Z}$ we let

$$P[w,g]^l = \bigoplus_{j=0}^{n-1} \Delta(\eta_w(j), l)\mathbf{P}_{c_w(j)}^{\deg g},$$

where Δ and c_w are as in Definition 3. The differential maps are $\delta^l_{P[w,g]^\bullet} = (\delta^l_{jk,w,g})_{0\leq j,k<n}$, where for each $l \in \mathbb{Z}$,

$$\delta^l_{jk,w,g} = \begin{cases} \mu_{w_{j+1}}\mathbf{Id}_{\deg g}, & \text{if } w_{j+1} \in \mathbf{Pa}_{>0}(\Lambda_0), \eta_w(j) = l \text{ and } k = j+1, \\ \mu_{w_j^{-1}}\mathbf{Id}_{\deg g}, & \text{if } w_j^{-1} \in \mathbf{Pa}_{>0}(\Lambda_0), \eta_w(j) = l \text{ and } k = j-1, \\ \mu_{w_n}F_g, & \text{if } w_n \in \mathbf{Pa}_{>0}(\Lambda_0), \eta_w(j) = l, j = n-1 \text{ and } k = 0, \\ \mu_{w_n^{-1}}F_g, & \text{if } w_n^{-1} \in \mathbf{Pa}_{>0}(\Lambda_0), \eta_w(j) = l, j = 0 \text{ and } k = n-1, \\ 0, & \text{otherwise,} \end{cases}$$

where F_g is the companion matrix of g.
We call the complex $P^\bullet[w,g]$ the **band complex** corresponding to the generalized band w.

Theorem 5. (H. GIRALDO & V-M, 2016) *Let* $\Lambda_0 = D(3\mathscr{R})^{1,2,2,2}$. *Then for all integers* $m \in \mathbb{Z}$, $w \in GSt(\Lambda_0)$ *(resp.* $w \in GBa(\Lambda_0)$ *and* $g \in \mathrm{ind}\,\Bbbk[x]$*), the complex* $T^m(P[w]^\bullet)$ *(resp.* $T^m(P[w,g]^\bullet)$*) is indecomposable in* $\mathcal{K}^b(\mathscr{P}_{\Lambda_0})$.

Here, T denotes the shifting functor, i.e. T shifts complexes one place to the left and changes the sign of the differential.

Sketch of the proof.

- Consider $\mathscr{M}(\Lambda_0)$ the set of maximal strings in Λ_0. We give the following linear ordering to $\mathscr{M}(\Lambda_0)$:

$$\zeta_0 \leq \tau_0\tau_1 \leq \zeta_1 \leq \tau_1\tau_2 \leq \zeta_2 \leq \tau_2\tau_0,$$

  and let $\mathscr{Y}(\Lambda_0) = (\mathscr{M}(\Lambda_0), \leq) \times \mathbb{Z}$.
- We order $\mathscr{Y}(\Lambda_0)$ anti-lexicographically as follows. For all $[u,l],[v,k] \in \mathscr{Y}(\Lambda_0)$ we have $[u,l] \preceq [v,k]$ if and only if $l \leq k$ or ($l = k$ and $u \leq v$).
- We define an involution σ over $\mathscr{Y}(\Lambda_0)$ (in the sense of (V. BONDARENKO, 1975)) as follows. For all $i \in \{0,1,2\} \mod 3$ and $m \in \mathbb{Z}$,

$$\sigma([\zeta_i, m]) = [\tau_{i+1}\tau_{i+2}, m].$$

□

Sketch of the proof (cont.).

- Consider the additive $\Bbbk$-category $\mathscr{S}(\mathscr{Y}(\Lambda_0),\Bbbk)$ of Bondarenko's matrix representations of $\mathscr{Y}(\Lambda_0)$ as explained in e.g. (V. BEKKERT & H. A. MERKLEN, 2003).
- We define a functor of additive categories

$$\mathbf{F}_{\Lambda_0} : \mathcal{K}^b(\mathscr{P}_{\Lambda_0}) \to \mathscr{S}(\mathscr{Y}(\Lambda_0),\Bbbk)$$

that identifies string and band complexes with indecomposables objects in $\mathscr{S}(\mathscr{Y}(\Lambda_0),\Bbbk)$.

□

Components of the Auslander-Reiten quiver of $\mathcal{K}^b(\mathscr{P}_{\Lambda_0})$

- We denote by $\Gamma(\mathcal{K}^b(\mathscr{P}_{\Lambda_0}))$ the Auslander-Reiten quiver of $\mathcal{K}^b(\mathscr{P}_{\Lambda_0})$.
- It follows from (W. Wheeler, 1994) and (D. Happel, B. Keller & I. Reiten, 2008) that if $\mathfrak{C}$ is a connected component of $\Gamma(\mathcal{K}^b(\mathscr{P}_{\Lambda_0}))$, then $\mathfrak{C}$ is of the form $\mathbb{Z}\mathbb{A}_\infty$.
- Thus, the component $\mathfrak{C}$ of $\Gamma(\mathcal{K}^b(\mathscr{P}_{\Lambda_0}))$ with a complex $C_0^\bullet$ lying on its boundary looks as in figure below.

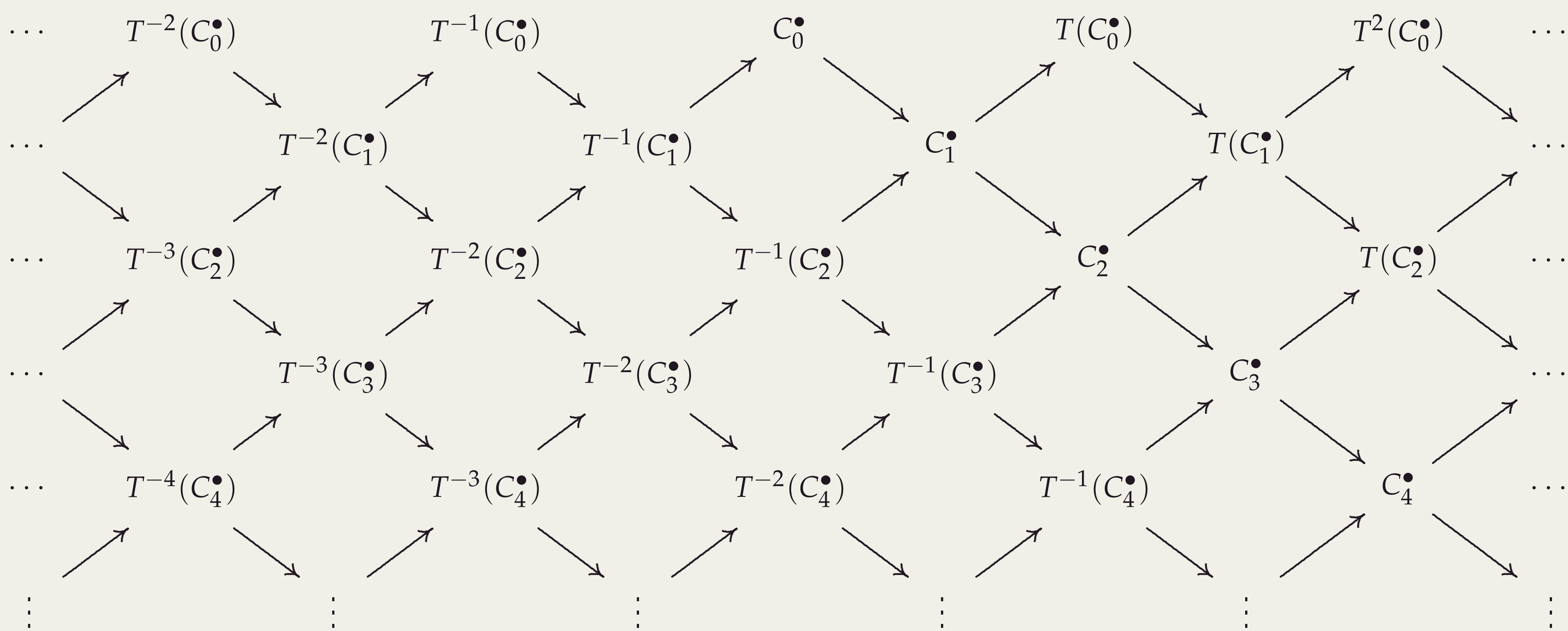


Figure 1: Component near $C_0^\bullet$

Let w be a generalized string representative for Λ_0. For all $k \geq 0$, we define a perfect complex $P_k[w]^\bullet$ as follows.

- We let $P_0[w]^\bullet = P[w]^\bullet$, and conveniently, we let $P_{-1}[w]^\bullet = 0^\bullet$.
- If $k \geq 1$, then $P_k[w]^\bullet$ is the complex such that

$$\mathrm{cone}(f^\bullet_{w,k-1}) = P_k[w]^\bullet \oplus T(P_{k-2}[w]^\bullet),$$

  where $f^\bullet_{w,k-1} : P_{k-1}[w]^\bullet \to P_{k-1}[w]^\bullet$ is the morphism in $\mathcal{C}^b(\mathscr{P}_{\Lambda_0})$ with $f^l_{w,k-1} = 0$ for all $l \neq \deg w$ and $f^{\deg w}_{w,k-1}$ is the morphism that sends isomorphically the top of $P_{k-1}[w]^{\deg w}$ to its socle.

Remark 6. *Observe that for all generalized band representatives w for Λ_0 and polynomials $g \in \mathrm{ind}\,\Bbbk[x]$, the construction above can be adjusted to obtain, for all $k \geq 0$, perfect complexes $P_k[w,g]^\bullet$ such that $P_0[w,g]^\bullet = P[w,g]^\bullet$.*

By using results from (W. Wheeler, 1994), (H. Giraldo & H.A. Merklen, 2009) and from (P. Webb, preprint), we obtain the following result.

Theorem 7. (H. Giraldo & V-M, 2016) *Let w be a generalized word for Λ_0.*

(i) *If w is a generalized string representative for Λ_0 of positive length and $\mathfrak{C}$ is the component of $\Gamma(\mathcal{K}^b(\mathscr{P}_{\Lambda_0}))$ containing $P[w]^\bullet$ as in Figure 1, then for all $k \geq 0$, $C_k^\bullet = P_k[w]^\bullet$.*

(ii) *If w be a generalized band representative for Λ_0, $g \in \operatorname{ind} \Bbbk[x]$ and $\mathfrak{B}$ is the component of $\Gamma(\mathcal{K}^b(\mathscr{P}_{\Lambda_0}))$ containing $P[w,g]^\bullet$ as in Figure 1, then for all $k \geq 0$, $C_k^\bullet = P_k[w,g]^\bullet$.*

Components of the Auslander-Reiten quiver of $\mathcal{K}^b(\mathscr{P}_{\Lambda_0})$

If w is a generalized string representative for Λ_0, then the component of the Auslander-Reiten quiver of $\mathcal{K}^b(\mathscr{P}_{\Lambda_0})$ containing the string complex $P_0[w]^\bullet = P[w]^\bullet$ looks like as in Figure 2.

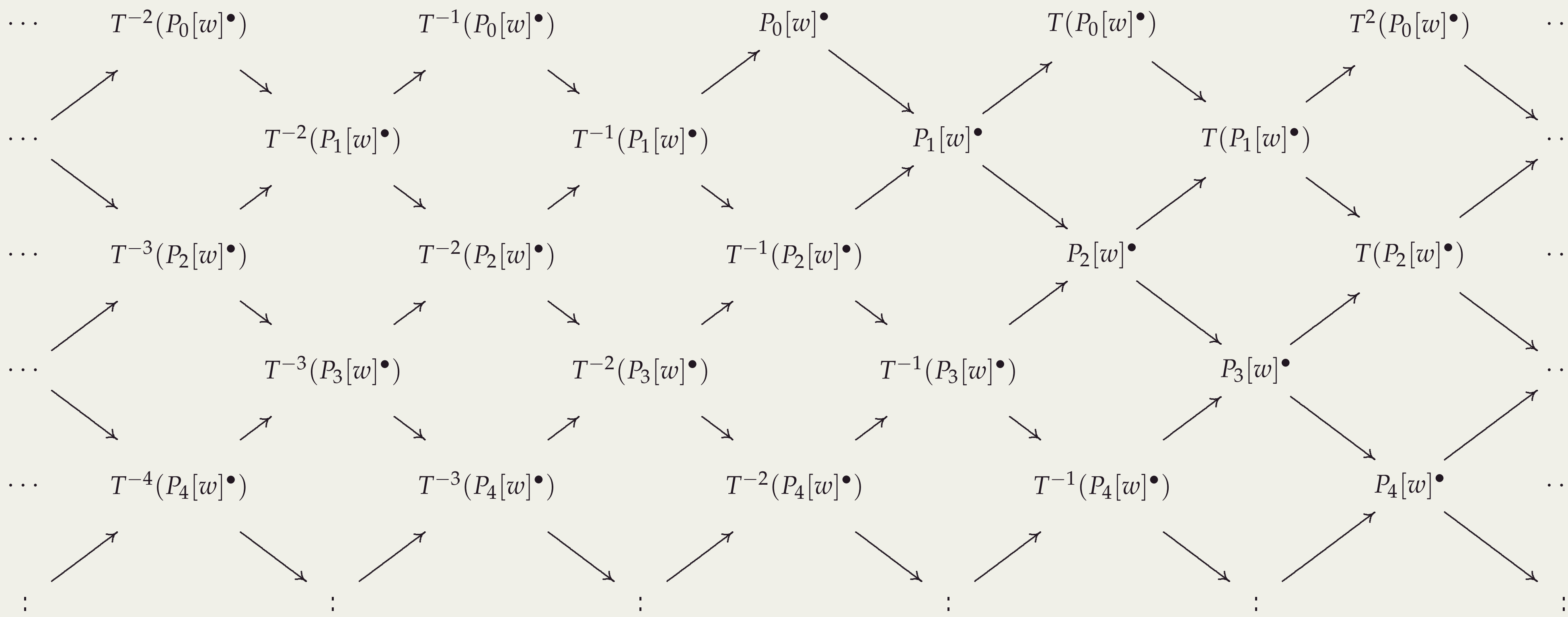


Figure 2: Component near $P[w]^\bullet = P_0[w]^\bullet$

Giraldo, H. and Vélez-Marulanda, J. A., **String and band complexes over certain algebra of dihedral type**. Algebr. Represent. Theory **19**(2), 419-433 (2016)

THANKS FOR YOUR ATTENTION!
MERCI DE VOTRE ATTENTION!
GRACIAS POR SU ATENCIÓN!
DANK FÜR IHRE AUFMERKSAMKEIT!