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**Title**: Syzygies and tensor products of modules

**Abstract**:

Given finitely generated nonzero modules $M$ and $N$ over a commutative Noetherian local ring $R$, the tensor product $M \otimes_R N$ of $M$ and $N$ typically has nonzero torsion. For example, if $R$ is the ring of formal power series $\mathbb{C}[\![x, y]\!]$ in indeterminates $x$ and $y$, and $I$ is the ideal of $R$ generated by $x$ and $y$, then $I \otimes_R I$ has nonzero torsion.

The assumption that the tensor product $M \otimes_R N$ is torsion-free influences the structure and vanishing of the modules $\mathsf{Tor}_i^R(M, N)$. In turn, the vanishing of $\mathsf{Tor}_i^R(M, N)$ imposes certain restrictions on the properties of $M$ and $N$. These connections made their first appearance in Auslander's 1961 seminal paper "Modules over unramified regular local rings". For example, if $M$ has finite projective dimension and the pair $(M, N)$ is Tor-independent, i.e., $\mathsf{Tor}_i^R(M, N) = 0$ for all $i \geq 1$, then it follows that $\operatorname{depth}(M) + \operatorname{depth}(N) = \operatorname{depth}(R) + \operatorname{depth}(M \otimes_R N)$. This equality is referred to as the *depth formula*, and has been generalized in several directions.

In this talk I will discuss an application of the depth formula and provide a partial answer to the following question that is motivated by results of Auslander, and Huneke and R. Wiegand: if $R$ is a complete intersection ring (e.g., a formal power series ring over a field modulo a regular sequence) and $M \otimes_R N$ is an $n$th syzygy module for some positive integer $n$, then must $M$ or $N$ be an $n$th syzygy module? The talk is based on a joint work with Greg Piepmeyer [Syzygies and tensor products of modules, Mathematische Zeitschrift, 276 (2014), no. 1, 457-468].