Standard Auslander-Reiten components of a Krull-Schmidt category

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- Want to describe maps in $\operatorname{mod} A$ between indecomposables.
- One introduces Auslander-Reiten quiver $\Gamma_{\operatorname{mod} A}$.
- In general, $\Gamma_{\operatorname{mod} A}$ describes maps not in $\operatorname{rad}^{\infty}(\operatorname{mod} A)$.

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3) (Ringel) Γ is preprojective or preinjective.

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2) *If Γ is regular and standard, then Γ is stable tube or $\Gamma \cong \mathbb{Z}\Delta$, where Δ a finite acyclic quiver.*

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2. In dual situation, f is *sink morphism.*

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REMARK. The above notion unifies almost split sequences in abelian categories and almost split triangles in triangulated categories.

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- *vertices*: the non-isomorphic indecomposables in $\mathcal{A}$.
- *arrows*: given X, Y, the number of arrows $X \to Y$ is $d_{X,Y}$.
- *translation*: if $X \longrightarrow Y \longrightarrow Z$ almost split, then $\tau Z = X$.

Objective

Question

1. How to decide a component of $\Gamma_{\mathcal{A}}$ is standard?

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2. Are there new types of standard components?
3. We consider these problems for components with a section.

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A connected full subquiver Δ of Γ is *section* if

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3. Δ is convex in Γ.

Example

Consider a *finite wing* as follows:

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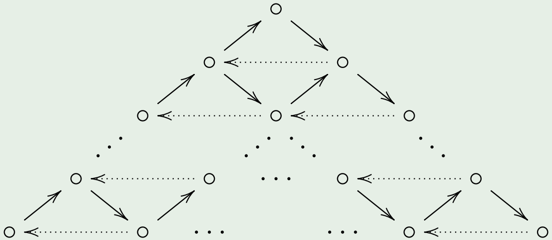

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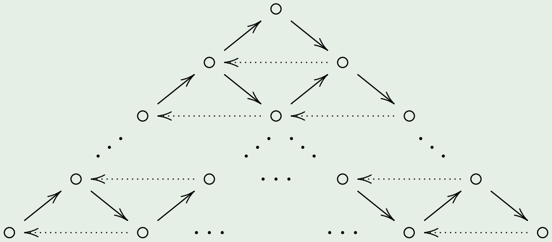

The two longest paths are sections.

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Remark

For $i \in \mathbb{Z}$, the subquiver (Δ, i) is section of $\mathbb{Z}\Delta$.

Let $\varDelta$ be acyclic quiver.

Construct translation quiver $\mathbb{Z}\varDelta$ in canonical way.

Remark

For $i \in \mathbb{Z}$, the subquiver $(\varDelta, i)$ is section of $\mathbb{Z}\varDelta$.

Notation

- $\mathbb{N}\Delta =< (x, i) \mid x \in \varDelta_0, i \in \mathbb{N} > \subseteq \mathbb{Z}\varDelta$.

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Remark

For $i \in \mathbb{Z}$, the subquiver (Δ, i) is section of $\mathbb{Z}\Delta$.

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- $\mathbb{N}\Delta = < (x, i) \mid x \in \Delta_0, i \in \mathbb{N} > \subseteq \mathbb{Z}\Delta$.
- $\mathbb{N}^{-}\Delta = < (x, -i) \mid x \in \Delta_0, i \in \mathbb{N} > \subseteq \mathbb{Z}\Delta$.

Example

The translation quiver $\mathbb{Z}\mathbb{A}_\infty$ is as follows:

$$\begin{tikzcd}
\cdots & \circ \arrow[dr] & & \circ \arrow[dr] & & \circ \arrow[dr] & & \circ \arrow[dr] & & \circ \arrow[dr] & & \circ & \cdots \\
& & \circ \arrow[ur] \arrow[dr] & & \circ \arrow[ur] \arrow[dr] & & \circ \arrow[ur] \arrow[dr] & & \circ \arrow[ur] \arrow[dr] & & \circ \arrow[ur] \arrow[dr] & & \\
\cdots & \circ \arrow[ur] & & \circ \arrow[ur] & & \circ \arrow[ur] & & \circ \arrow[ur] & & \circ \arrow[ur] & & \circ & \cdots \\
& \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots &
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then $\mathbb{N}\mathbb{A}_\infty^+$ is follows:

$$\begin{tikzcd}
\circ \arrow[dr] & & \circ \arrow[ll, dotted] \arrow[dr] & & \circ \arrow[ll, dotted] \arrow[dr] & & \cdots & & \\
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& & \circ \arrow[ur] \arrow[dr, dotted] & & \circ \arrow[ll, dotted] \arrow[ur] \arrow[dr, dotted] & & \circ \arrow[ll, dotted] \arrow[dr, dotted] & & \cdots \\
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then $\mathbb{N}^-\mathbb{A}_\infty^-$ is as follows:

$$\begin{tikzcd}[column sep=small]
 & & \cdots & \circ \arrow[dr] & & \circ \arrow[ll, dotted] \arrow[dr] & & \circ \arrow[ll, dotted] \\
 & \cdots & \circ \arrow[ur] \arrow[dr] & & \circ \arrow[ll, dotted] \arrow[ur] \arrow[dr] & & \circ \arrow[ll, dotted] \arrow[ur] & \\
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Properties of components with sections

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Then Γ is standard $\Leftrightarrow \mathrm{Hom}_{\mathcal{A}}(\Delta^{+}, \Delta^{-}) = 0$.

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Γ is standard $\Leftrightarrow$ the quasi-simple objects are orthogonal bricks.

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$\mathrm{proj}(Q)$: additive category of the P_x, $x \in Q_0$.

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where $P_0, P_1 \in \mathrm{proj}(Q)$.

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$\mathrm{rep}^+(Q)$ is Hom-finite, hereditary, abelian.

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$\Delta^- = \emptyset$ and Δ^+ no left-∞ path.

$\mathrm{add}(\Delta) \cong kQ^{\mathrm{op}}$ and $\mathrm{Hom}(\Delta^+, \Delta) = 0$.

Theorem

Let Q connected, infinite, strongly locally finite.

Regular components

Theorem

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1. *The regular components of $\Gamma_{\mathrm{rep}^+(Q)}$ are wings or $\mathbb{Z}\mathbb{A}_\infty, \mathbb{N}\mathbb{A}_\infty^+, \mathbb{N}^-\mathbb{A}_\infty^-$.*

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2. *The regular components are all standard $\Leftrightarrow$ Q of infinite Dynkin types $\mathbb{A}_\infty, \mathbb{A}_\infty^\infty, \mathbb{D}_\infty$.*

Theorem

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Infinite Dynkin case

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1. $\Gamma_{\mathrm{rep}^+(Q)}$ *has at most four components, at most two regular, all standard.*

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2. *Wings,* $\mathbb{Z}\mathbb{A}_\infty, \mathbb{N}\mathbb{A}_\infty^+, \mathbb{N}^-\mathbb{A}_\infty^-$ *all appear in this setting.*

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3. *Q of infinite Dynkin type $\Rightarrow \Gamma_{D^b(\mathrm{rep}^+(Q))}$ has at most 3 components up to shift, all standard.*

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