The Invariant Theory of Artin-Schelter Regular Algebras: A Survey

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Gauss showed that when the symmetric group S_n acts as permutations of $\{x_1, \ldots, x_n\}$, the ring of invariants $k[x_1, \ldots, x_n]^{S_n}$ is a polynomial ring in the elementary symmetric functions. Classical invariant theory considers the structure of the ring of invariants A^G obtained when a finite group G of $n \times n$ matrices acts linearly on $A = k[x_1, \ldots, x_n]$, for k a field of characteristic 0. We consider generalizations of these results in the setting that A is an Artin-Schelter regular algebra, and G is a finite group, acting linearly on the generating set of A. More generally, we consider actions on A by a finite dimensional Hopf algebra H. We present some answers to some basic questions on the the structure of A^G, including when it is AS-regular, or AS-Gorenstein, or a "complete intersection". We consider generalizations of the dimension 2 case, when a finite subgroup of $SL_2(k)$ acts on $k[u, v]$. As a commutative Artin-Schelter regular ring must be a commutative polynomial ring, these results can be viewed as extensions of the classical theory.