

RECOLLEMENTS FROM PARTIAL TILTING COMPLEXES

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ABSTRACT. From [DG], [Mi] and [J] it is known that every compact object Q of the derived category $\mathcal{D}(B)$ of a dg-algebra gives rise to a recollement of triangulated categories of the form

$$\begin{array}{ccc}
 & & Q \overset{L}{\otimes}_D - \\
 & \overset{i^*}{\curvearrowright} & \\
 (*) \quad Q^\perp & \xrightarrow{i_* = inc} & \mathcal{D}(B) \xrightarrow{\mathbb{R}\mathrm{Hom}_B(Q, -) \cong P \overset{L}{\otimes}_B -} \mathcal{D}(D) \\
 & \underset{i^!}{\curvearrowleft} & \\
 & & \mathbb{R}\mathrm{Hom}_D(P, -)
 \end{array}$$

with $P = \mathbb{R}\mathrm{Hom}_B(Q, B)$.

Following [NS] we show that the left hand term of the recollement above is equivalent to the derived category of a dg algebra C linked to B by a homological epimorphism and we study the TTF triple associated to the recollement. A particular case of (*) gives a generalization of the Morita-type theorem proved by Rickard in [R].

As an application we obtain the same result as in [BMT] but with much weaker assumptions. Moreover, our setting generalizes to the case of infinitely generated n -tilting modules, the results proved recently by [CX] for 1-tilting modules. Finally we characterize when the left hand term of (*) is exactly a ring, introducing the concept of “generalized universal localization”.

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