Manipulating Smith-Purcell Emission with Babinet Metasurfaces

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Swift electrons moving closely parallel to a periodic grating produce far-field radiation of light, which is known as the Smith-Purcell effect. In this letter, we demonstrate that designer Babinet metasurfaces composed of *C*-aperture resonators offer a powerful control over the polarization state of the Smith-Purcell emission, which can hardly be achieved via traditional gratings. By coupling the intrinsically nonradiative energy bound at the source current sheet to the out-of-plane electric dipole and in-plane magnetic dipole of the *C*-aperture resonator, we are able to excite cross-polarized light thanks to the bianisotropic nature of the metasurface. The polarization direction of the emitted light is aligned with the orientation of the *C*-aperture resonator. Furthermore, the efficiency of the Smith-Purcell emission from Babinet metasurfaces is significantly increased by 84%, in comparison with the case of conventional gratings. These findings not only open up a new way to manipulate the electron-beam-induced emission in the near-field region but also promise compact, tunable, and efficient light sources and particle detectors.

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Metasurfaces, two-dimensional metamaterials, have recently emerged as a new frontier of science because they provide large degrees of freedom to control over the propagation of light [1]. Metasurfaces allow us to tailor the phase, amplitude, polarization, and ray trajectory [2–4] of light based on a single layer of engineered structures or meta-atoms [5–7]. Conventional optical devices usually rely on phase accumulation over a long optical path due to the inherently weak interaction of light and matter. Metasurfaces, in contrast, provide an elegant way to overcome the constraint by designing suitable subwavelength meta-atoms and arranging their spatial distributions in a prescribed manner to regulate the local light-matter interactions. This new methodology has enabled numerous novel optical phenomena and devices on a planar platform, including unidirectional coupling of surface plasmon polaritons [8–10], information processing [11], spin-orbit manipulation [12,13], highly efficient holography [14], and an ultrathin invisibility cloak [15]. Metasurfaces have also been employed in nonlinear optics, where they provide promising approaches to control the nonlinearity phase [16] and amplify the efficiencies of nonlinear processes [17,18].

The interaction of swift electrons with a material can generate far-field electromagnetic radiation that is coherent to the evanescent field associated with the moving electrons. This effect is well known as electron-induced emission [19]. The analysis of various interactions between electrons and materials is a substantial source of inspiration for advanced electron microscopy, including electron energy-loss spectroscopy, and cathodoluminescence emission, including transition radiation, Cherenkov radiation, and diffraction radiation. Recent breakthroughs in artificially engineered metamaterials and nanotechnology manifest new opportunities to tailor the interaction of the electron with matter [20]. For example, reversed Cherenkov radiation has been proposed and directly observed [21] in left-handed metamaterials, which allow easy separation of the backward radiation of light from the incoming particles [21,22]. Swift electrons can excite surface plasmon polaritons on a metal film, which can be thereafter transformed into Cherenkov radiation with enhanced intensity [23]. Other unusual Cherenkov emissions have also been reported for charged particles moving within or in the vicinity of anisotropic metamaterials [24–26], photonic crystals [27], and dielectric clusters [28]. However, to the best of our knowledge, so far little work has discussed how to manipulate the polarization state of the electron-induced emission.

In this Letter, we show that the polarization state of Smith-Purcell emission, a well-known example of diffraction radiation excited by electron beams [19,29], can be effectively controlled with Babinet metasurfaces. Theoretical analysis of the Smith-Purcell emission from periodically corrugated metasurfaces is first performed, which reveals the unique approach to generating cross-polarized emission. *C*-aperture Babinet metasurfaces resonant at terahertz frequencies are then designed to achieve required cross-coupled electric and magnetic dipoles. Numerical results demonstrate that the polarization angle of the radiated light is determined by the orientation of the *C*-aperture resonators. Subsequently, Stokes parameters are employed to accurately characterize the polarization state of the Smith-Purcell emission. Last, the emitted light from the proposed metasurfaces exhibits much higher intensity than that from conventional metallic gratings, indicating that the resonant

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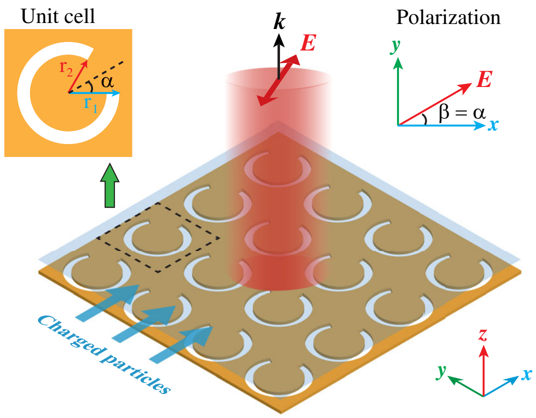

FIG. 1. Schematic of the Smith-Purcell emission from a Babinet metasurface. A uniform sheet of charged particles moves closely parallel to the metasurface along the $+x$ axis. The Babinet metasurface generates linearly polarized radiation, propagating along the surface normal in the $+z$ direction. The direction of the electric field has an angle β with respect to the $+x$ axis, which is determined by the azimuthal rotating angle α of the C-aperture resonators.

nature of metasurfaces can significantly enhance the radiation efficiency. These results provide a new strategy to develop a compact, tunable, and high power terahertz source based on the Smith-Purcell emission [30,31], polarization-sensitive detectors, and other novel photonic devices.

The concept of metasurface-mediated Smith-Purcell emission is illustrated in Fig. 1. A uniform sheet of charged particles moves closely above a metasurface, and the velocity of particles is parallel to the periodicity of metasurface. The induced current on the metallic surface will generate far-field radiation if the periodicity is properly designed. Traditional one-dimensional gratings generate far-field radiation with the magnetic field always polarized in one direction. This constraint can be eliminated by using a Babinet metasurface comprised of C apertures. The polarization angle (β) of the radiated light can be steered by controlling the orientation of the C-aperture resonator (α), and in fact β is identical to α.

We first review the physical mechanism of the Smith-Purcell emission. Assume a metasurface sits on the xy-plane at $z = 0$ and the charged particles move with a uniform velocity v_0 along the x axis above the metasurface at $z = z_0$. The current density can be expressed as $\bar{J}(x, z, t) = \hat{x}qv_0\delta(z - z_0)\delta(x - v_0t)$. Here, q is the charge density distribution per unit length in the y direction. After Fourier transform, the current density in the frequency domain is then given by

$$\bar{J}(x,z,\omega) = \frac{1}{2\pi}\int dt\bar{J}(x,z,t)e^{i\omega t} = \hat{x}I_0\delta(z-z_0)e^{ik_x x}, \quad (1)$$

where $k_x = \omega/v_0$ and $I_0 = q/2\pi$. When $v_0 > c$ with c being the speed of light in the surrounding medium, plane waves can be radiated into the free space without using any periodic structure, which is known as Cherenkov radiation. In contrast, if $v_0 < c$, the fields associated with the incident current are nonradiative, exponentially decaying away from the current sheet. If we place a periodic structure close to the current sheet, the Smith-Purcell emission occurs, generating far-field propagating waves. In the Smith-Purcell emission, the surface current expressed by Eq. (1) generates pure evanescent fields that can be described by a magnetic vector potential $\bar{A} = \hat{x}(\mu_0 I_0/2\gamma)e^{-\gamma|z-z_0|+ik_x x}$, where $\gamma = \sqrt{k_x^2 - k_0^2}$, and μ_0 and k_0 are the permeability and the wave number associated with the free space, respectively. Hence, the evanescent field induced by the current in the region $0 < z < z_0$ can be written as

$$\bar{E}_i = -(\hat{x}i\gamma + \hat{z}k_x)\frac{I_0}{2\omega\varepsilon_0}e^{\gamma(z-z_0)+ik_x x},$$
$$\bar{H}_i = \hat{y}\frac{I_0}{2}e^{\gamma(z-z_0)+ik_x x}. \quad (2)$$

In the upper half-space above the metasurface, we can expand the reflected fields in terms of Floquet modes as

$$\bar{H}_r = \sum_{m,n}\bar{H}_{mn}e^{i(k_x+2m\pi/p)x+i(2n\pi/p)y+ik_{zmn}z}, \quad (3)$$

where $k_{zmn} = \sqrt{k_0^2 - (k_x + 2m\pi/p)^2 - (2n\pi/p)^2}$ is the wave number in the z direction, p is the periodicity in both x and y directions, and $\bar{H}_{mn}$ is the magnetic field vector of each diffraction order. For periodic structures with subwavelength lattice constants ($p < \lambda_0$), any Floquet mode with either positive m or nonzero n corresponds to an evanescent wave that decays exponentially along the z axis, where λ_0 is the wavelength of light in the surrounding medium. Note that n has to be zero for far-field radiation. This means the transverse component of wave vector k_y vanishes. Consequently, periodically corrugated gratings with subwavelength inclusions only generate far-field radiation propagating in the xz plane when charged particles move along the x axis. By substituting $k_{zmn}^2 > 0$ into Eq. (3), we can obtain the necessary condition for the Smith-Purcell emission

$$\frac{p}{|m|}\left(\frac{c}{v_0} - 1\right) \le \lambda_0 \le \frac{p}{|m|}\left(\frac{c}{v_0} + 1\right), \quad \text{with} \quad m < 0 \quad \text{and} \quad n = 0. \quad (4)$$

In particular, when the periodicity is carefully designed to $p = -mv_0\lambda_0/c$, the transverse components of the wave vector vanish ($k_x, k_y = 0$), and therefore the emitted light propagates along the z axis. For simplicity and without loss of generality, we restrict the following discussion to this case.

In this section, we discuss how to manipulate the polarization of the Smith-Purcell emission with Babinet metasurfaces. According to Eq. (2), one can find that the incident field is a pure transverse magnetic (TM) wave, in which the magnetic field contains only a y component (H_{yi}) while the electric field contains both x and z components (E_{xi} and E_{zi}). To generate far-field radiation with the electric field polarized in the y direction, a straightforward way is inducing an array of y-oriented electric dipoles (p_y) or x-oriented magnetic dipoles (m_x), based on their radiation characteristics [32]. Nevertheless, the excitation of p_y directly by E_{xi} is forbidden because they are orthogonal to each other. The same issue holds true for the magnetic dipole m_x and incident magnetic field H_{yi}. Therefore, directly coupling the electric (magnetic) field to the electric (magnetic) dipole moment is impractical. On the other hand, the cross-coupling between the electric (magnetic) field and the magnetic (electric) dipole moment is not prohibited. This scenario can be realized by using an anisotropic Babinet metasurface. Let us consider a metasurface consisting of C-shaped resonators as illustrated in Fig. 2(a). For a C-ring resonator (left panel) under an illumination of E_{xi}, the in-plane electric dipole p_x and the out-of-plane magnetic dipole m_z will be excited. Since no z-oriented electric dipole and y-oriented magnetic dipole exist in the C-ring resonator, the incident fields of E_{zi} and H_{yi} cannot contribute to excite any resonance mode. Therefore, cross-polarization emission is forbidden in this case for C-ring resonators. However, the situation is completely different for the complementary structure. According to the Babinet principle, the electromagnetic responses of the complementary structure, *i.e.*, a C-aperture resonator (right panel), can be described by the fictitious out-of-plane electric dipole p_z and in-plane magnetic dipole m_x. In particular, p_z can be excited by the external field of E_{zi}. Since the electric and magnetic resonances are coupled in the C-aperture metasurface, p_z can further induce the in-plane magnetic dipole m_x. Subsequently, the far-field radiation of cross polarization is achieved. This approach for cross-polarized emission can also be understood as the near-field excitation of the bianisotropic response [33]. Moreover, the orientation of the in-plane magnetic dipole can be easily changed by rotating the C-aperture resonator about the z axis, thus enabling flexible control over the emission polarization.

A proof-of-concept metasurface for emission polarization control is designed in the terahertz regime. The outer and inner radii of the C-aperture resonator are $r_1 = 20$ μm and $r_2 = 15$ μm, respectively. The central angle of the arc is 325°, and the periodicity is $p = 50$ μm. Gold is chosen as the metallic material with a conductivity of 4×10^7 S/m and a thickness of 0.2 μm. The simulated reflection spectra of the Babinet metasurface at normal incidence are plotted in Fig. 2(b), showing a strong resonance at the frequency of 1.5 THz under transverse electric (TE) polarization (with the E_y component) yet no resonance under TM polarization (with the H_y component). However, the situation is distinct for oblique incidence of TM waves. Since the incident electric field contains a z component, the out-of-plane electric dipoles p_z can be excited, and then it induces the in-plane magnetic dipole m_x. Such coupling between the electric and magnetic responses leads to the polarization conversion, which can be described by the cross-polarization reflection coefficient r_{12} [Fig. 2(c)]. The subscripts 1 and 2

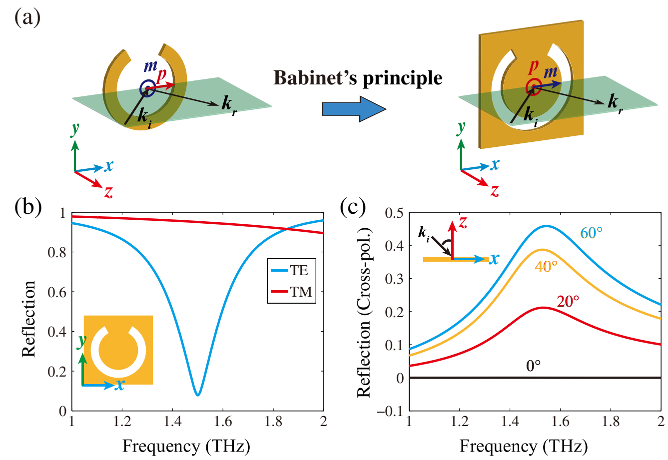

FIG. 2. The responses of the C-ring and C-aperture metasurfaces under illumination of propagating waves. (a) Illustration of the dipolar behaviors of a C-ring resonator and of its complementary structure, namely, a C-aperture resonator. (b) Reflection coefficients of the C-aperture metasurface at normal incidence. TE and TM modes correspond to the incident fields of E_y and H_y, respectively. (c) Cross-polarization reflection coefficients (r_{12}) of the C-aperture metasurface at oblique incidence of TM waves.

represent the TE and TM modes, respectively. As the incident angle increases, a significant amount of the incident light can be converted to the cross-polarized reflection. This phenomenon can also be explained by the mechanism of extrinsic chirality [34]. At oblique incidence, the mirror symmetry of the structure with respect to the incident plane is broken, leading to a discrepancy of the two spin states in reflection [34]. As a comparison, the electromagnetic responses of the C-ring metasurface are discussed in the Supplemental Material [35].

To demonstrate the ability to steer the polarization of Smith-Purcell emission with the aforementioned Babinet metasurface, full-wave electromagnetic simulations have been performed using the commercial software of COMSOL MULTIPHYSICS. A sheet of surface current is placed 10 μm above the metasurface with particle moving velocity $v_0 = c/4$ and current density $I_0 = 1$ A/m. Four unit cells of C-aperture resonators are constructed in a three-dimensional model with the periodic boundary conditions. Under these conditions, the reflected Smith-Purcell emission at 1.5 THz propagates along the normal vector of the surface ($+z$ direction). The simulated field distributions of the Smith-Purcell emission are shown in Fig. 3(a). All the field images are the xz-plane snapshots of the electric fields at the position of $y = 0$ (the center of the C-aperture resonator). There are two rows of little ridges in the middle of the images in Fig. 3. The top one represents the bound surface waves at the location of the surface current, while the bottom one corresponds to the locally enhanced field of the metasurface. One can see that the emitted light exhibits a purely x-polarized state ($E_y = 0$) when the C-aperture resonator is oriented in the x direction ($\alpha = 0°$). The orthogonal polarization ($E_x = 0$) is obtained by rotating the resonator

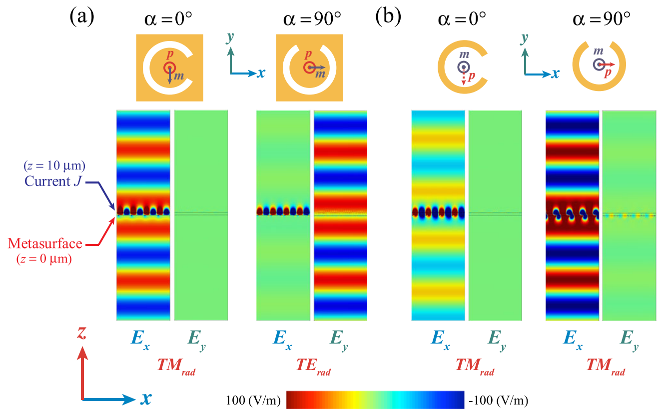

FIG. 3. Electric field distributions of the Smith-Purcell emission from (a) C-aperture and (b) C-ring metasurfaces. A sheet current with the parameters $v_0 = c/4$, $I_0 = 1$ A/m described by Eq. (1) acts as the source in simulation. The sheet current is located at 10 μm above the metasurface.

90 degrees about the z axis ($\alpha = 90°$). This clearly shows that the C-aperture metasurface can be used as the polarization modulator of Smith-Purcell emission. To better verify the behaviors of the microscopic dipoles, the Smith-Purcell emission in a complementary C-ring metasurface has also been investigated as a comparison [Fig. 3(b)]. When $\alpha = 90°$, the in-plane x-oriented electric dipole can be directly excited by the E_{xi} component of the incident electric field. Hence, efficient far-field generation of x-polarized light occurs. In the case of $\alpha = 0°$, the orientation of the fundamental electric dipole mode of is changed to the y direction (dashed arrow), which is orthogonal to the incident electric fields E_{xi} and E_{zi}. The excitation of this resonant mode is forbidden, and no cross-polarized light is radiated. In addition, the C-ring metasurface radiates the electric field with relatively weak intensity for $\alpha = 0°$, in which case the structure is off resonance.

We now consider the influence of the rotating angle α on the polarization of the Smith-Purcell emission based on the widely used Stokes parameters [36]. The electric field of the emitted light can be expressed as $\bar{E}_{\rm rad} = E_1 e^{i\phi_1}\hat{x} + E_2 e^{i\phi_2}\hat{y}$, where E_1 and E_2 are the amplitudes of the two components E_x and E_y, and ϕ_1 and ϕ_2 are the corresponding phases, respectively. The Stokes parameters are given by

$$S_0 = E_1^2 + E_2^2, \quad S_1 = E_1^2 - E_2^2,$$
$$S_2 = 2E_1E_2\cos(\phi_2 - \phi_1), \quad S_3 = 2E_1E_2\sin(\phi_2 - \phi_1). \quad (5)$$

The polarization state of a light wave can be characterized by introducing two parameters: the orientation angle β and the ellipticity angle τ of the polarization ellipse [Fig. 4(a)]. These two angles can be derived from the Stokes parameters

$$\tan 2\beta = S_2/S_1, \text{ and } \sin 2\tau = S_3/S_0, \quad (6)$$

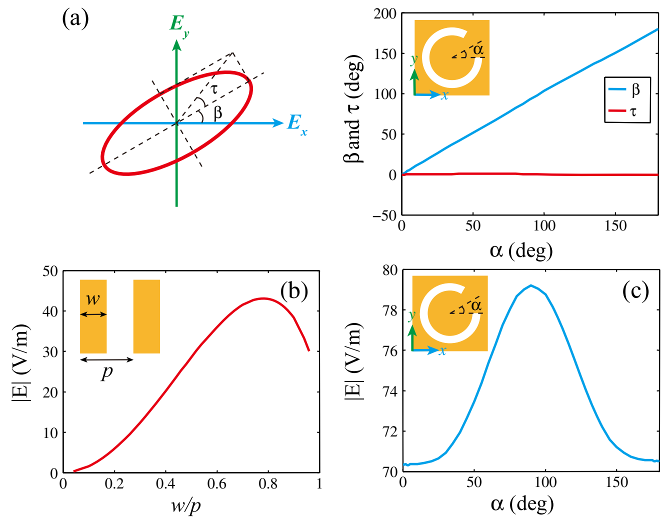

FIG. 4. (a) Illustration of the polarization ellipse (left panel) and the calculated orientation angle β and ellipticity angle τ of the emitted light (right panel). (b),(c) The amplitude of Smith-Purcell emission from 1D metallic gratings and C-aperture metasurfaces, respectively. The periodicity of the grating is the same as the metasurface ($p = 50$ μm).

with $0 \le \beta \le \pi$ and $-\pi/4 \le \tau \le \pi/4$. The dependence of β and τ on the rotating angle α is plotted in Fig. 4(a), which is extracted from the averaged electric fields in the xy-plane 140 μm above the surface current (at $z = 150$ μm). Interestingly, the orientation angle β is identical to the rotating angle α while the ellipticity angle τ is kept near 0, indicating that the radiated light is linearly polarized and its orientation angle can be continuously steered by rotating the C-aperture resonators. Such an attractive performance is attributed to our unique strategy of exciting the in-plane magnetic dipole by the out-of-plane electric field E_{zi}. Thus, the magnetic dipole can always be induced no matter how its orientation changes as the C aperture rotates. Moreover, the out-of-plane electric dipole p_z has no contributions to the far-field radiation in the normal direction ($+z$) so that the polarization state is only determined by the induced in-plane magnetic dipole. This feature ensures the continuous manipulation of the polarization with only one controlling variable.

Another important metric for Smith-Purcell emission is the radiation efficiency. We compare the radiation efficiency of the Babinet metasurface with that of the traditional 1D metallic grating. For a fair comparison, the periodicity and thickness of the metallic grating are also set to 50 and 0.2 μm, respectively. Figure 4(b) plots the amplitude of the radiated electric field versus the width (w) of the grating. The maximum efficiency is achieved when $w/p = 0.78$ with the electric field amplitude being 43.1 V/m. The radiation amplitude of the Babinet metasurface as a function of the rotating angle α is shown in Fig. 4(c). It is interesting to note that the radiation amplitude is always much larger than that of the 1D grating, no matter how the C-aperture resonator rotates.

The maximum amplitude of 79.2 V/m is reached when $\alpha = 90°$, which is 84% larger than that of the grating. We have performed additional simulations to investigate the influence of the geometry of the C-aperture resonator on the emission properties [35]. The results show that high radiation efficiency can be achieved for a quite broad range of geometric parameter variations.

In conclusion, we show that a Babinet metasurface made of C-aperture resonators can effectively modify the coupling channel of Smith-Purcell emission, enabling continuous control over the polarization direction of the radiated light. This is achieved by exciting the fictitious in-plane magnetic dipole moment via the out-of-plane component of the evanescent field associated with the moving electrons. Compared with conventional 1D gratings, the proposed metasurface can increase the radiated efficiency up to 84%, thus promising high-efficiency electron-induced emission with controlled polarization states. Our findings offer a versatile platform to extract and explore the near-field energy carried by relativistic electron beams and manifest promising applications in radiation generators and particle detectors. In addition, due to the scaling properties of Maxwell's equations, the design principle proposed here is universal and can be easily extended to other frequencies. Since polarization is a fundamental property of light, we expect our work can significantly promote the advance in novel electron microscopy techniques to probe material properties, electron-beam-based photonic technologies (e.g., free-electron lasers), as well as the emerging area of photonic spin-orbit interactions based on structured interfaces [37,38].

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[35] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.117.157401 for a detailed discussion of the behavior of the *C*-ring metasurface in Smith-Purcell emission and the dependence of its emission performance on the geometric parameters.

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