

HW1, extra-credit problem: Let  $f_1, \dots, f_k$  be a finite collection of <sup>homogeneous</sup>  $G$ -invariant elements generating  $\mathbb{C}[V] \cong \mathbb{C}[V]^G$ . The common set of zeroes of  $f_1, \dots, f_k$  is  $\{0\}$  b/c the  $G$ -invariant elements separate the orbits. So the preimage of  $0 \in \mathbb{C}^k$  under  $V \xrightarrow{(f_1, \dots, f_k)} \mathbb{C}^k$  is  $\{0\}$ . It follows that this morphism is finite. Therefore all fibers are finite. The morphism is also  $G$ -invariant. It follows that all  $G$ -orbits are finite. But  $G \subseteq GL(V)$ , hence  $G^0 = \{1\}$  and  $G$  is finite.