19. KZ functor I

Problem 19.1. Let $M_1, M_2$ be $D_X$-modules that are coherent sheaves. Show that
$$\dim \text{Hom}_{D_X}(M_1, M_2) < \infty.$$ 

Exercise 19.1. Let $M$ be an $H_c$-module with locally nilpotent action of $\mathfrak{h}$. Show that $M$ is finitely generated iff the action of $h$ on $M$ is locally finite and all generalized eigen-subspaces are finite dimensional.

Problem 19.2. Show that $\text{Ext}^i(\Delta(E), \nabla(E')) = \mathbb{C}$ if $E = E', i = 0$, and 0 else. Moreover, show that if $\text{Ext}^1(\Delta(E), M) = 0$ for all $E$, then $M$ is $\nabla$-filtered, i.e., admits a filtration with successive quotients $\nabla(E')$.

Problem 19.3. A $\Delta$-filtered object $M$ is projective iff $\text{Ext}^1(M, \Delta(E)) = 0$ for all $E$.

Problem 19.4. (1) Show that the double centralizer property is equivalent to $\text{Ext}^1(M, P) = 0$ for any projective $P$ and $M \in \mathcal{O}_\text{tor}$.

(2) Use the naive duality to show that $\pi$ is fully faithful on injectives.

(3) Show that $\pi$ has left adjoint $\pi^!$ and that $\pi \circ \pi^!$ is the identity on the image of $\pi$.

\footnote{This exercise and the next problem also appeared last time.}