PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

12. CM systems and quantum mechanics

Exercise 12.1. Show that the trajectories for \( H = \frac{1}{2} \text{tr}(Y^2) \) on \( R = T^* \text{Mat}_n(\mathbb{C}) \) are of the form \( (X - tY, Y) \).

Problem 12.1. Prove part (2) of the main theorem (integration of the CM system) in Lecture 11.

Problem 12.2. Check that the symplectic forms on \( \mu^{-1}(E)/\tilde{G} \) and \( \mu^{-1}(O)/G \) (see the notation in the lecture notes) are the same.

Exercise 12.2. Show that the algebra \( D_h(X_0) \) is a deformation of \( \mathbb{C}[T^*X_0] \) compatible with the usual bracket there. Hint: how does the sheaf \( D_h(X_0) \) on \( X_0 \) behave under étale base changes?

Exercise 12.3. Let \( \mathcal{A}_h \) be a \( \mathbb{Z}_{>0} \)-graded \( \mathbb{C}[h] \)-algebra with \( h \) being of positive degree. Let \( \mathcal{A}'_h \) be the \( h \)-adic completion of \( \mathcal{A}_h \). Explain how to recover \( \mathcal{A}_h \) back from \( \mathcal{A}'_h \) using some natural action of \( \mathbb{C}^\times \) on \( \mathcal{A}'_h \).

Exercise 12.4. Let \( X_0 \) be a smooth affine variety acted freely by a finite group \( \Gamma \). Equip \( D_h(X_0) \) with a natural \( \Gamma \)-action by \( \mathbb{C}[h] \)-algebra automorphisms and then identify \( D_h(X_0)^\Gamma \) with \( D_h(X_0/\Gamma) \).

\footnote{This exercise and the next problem already appeared in Pset 11}