Prop. 5: Let $Y_1, Y_2 \subset X$ be closed G-invariant subvarieties. If $Y_1 \cap Y_2 = \phi$, then $\pi(Y_1) \cap \pi(Y_2) = \phi$.

Proof: $Y_1 \cap Y_2 = \phi \Rightarrow \mathbb{C}[Y_1 \cup Y_2] = \mathbb{C}[Y_1] \oplus \mathbb{C}[Y_2]$. Note that $Y_1 \sqcup Y_2$ is a closed subvariety on X. So the restriction map $\varphi : \mathbb{C}[X] \to \mathbb{C}[Y_1 \sqcup Y_2]$ is surjective. In particular, $\exists f \in \mathbb{C}[X] \mid \varphi(f) = (1,0)$, i.e. $f|_{Y_1} = 1$, $f|_{Y_2} = 0$. The condition $\pi(Y_1) \cap \pi(Y_2) = \phi$ will follow if we show that there's $\underline{f} \in \mathbb{C}[X/G]$ w. $\underline{f}|_{\pi(Y_1)} = 1$, $\underline{f}|_{\pi(Y_2)} = 0 \Leftrightarrow \underline{f} \circ \pi|_{Y_1} = 1$, $\underline{f} \circ \pi|_{Y_2} = 0$. By the construction of π, the functions of the form $\underline{f} \circ \pi \in \mathbb{C}[X]$ (for some $\underline{f} \in \mathbb{C}[X/G]$) are precisely the G-inv't functions. So our goal is to find $\tilde{f} \in \mathbb{C}[X]^G$ with $\tilde{f}|_{Y_1} = 1$, $\tilde{f}|_{Y_2} = 0$. We claim that $\tilde{f} := \alpha(f)$ works. Indeed, φ is G-equivariant and $(1,0) \in \mathbb{C}[Y_1 \sqcup Y_2]$ is G-invariant. By (3) of Lem 1. $\varphi \circ \alpha = \alpha \circ \varphi$. Plug f. Then $\varphi(f) = (1,0)$ is G-invariant so $\alpha \circ \varphi(f) = \varphi(f)$. So $\alpha(f)|_{Y_1} = 1$, $\alpha(f)|_{Y_2} = 0$. $\square$