DAY 2 EXERCISES

1. QUANTIZATION OF ALGEBRAS

Exercise 1.1. Show that the bracket on \( gr \mathcal{A} \) is well-defined and is a Poisson bracket.

Exercise 1.2. Show that the product in \( \mathcal{A} / / _{\lambda} \mathcal{G} \) is well-defined.

Remark: You have to show both that the product is well-defined, and that \( \mathcal{A} / / _{\lambda} \mathcal{G} \) is closed under the product!

2. QUANTIZATION OF SHEAVES

Exercise 2.1 (Exercise 2.1 in the notes). Let \( \mathcal{A} \) be a (complete and separated) quantization of \( \mathcal{A} \). Show that if \( \mathcal{A} \) is Noetherian so is \( \mathcal{A} \).

Hint: Let \( \mathcal{I} \) be a left ideal of \( \mathcal{A} \). Pick homogeneous generators \( \overline{a}_1, \ldots, \overline{a}_n \) of \( gr \mathcal{I} \), and pick lifts \( a_1, \ldots, a_n \in \mathcal{I} \). Show that \( a_1, \ldots, a_n \) are generators of \( \mathcal{I} \).

Exercise 2.2. Let \( \mathcal{A} \) be a filtered algebra, complete and separated. Assume that \( gr \mathcal{A} \) is Noetherian. Then any left ideal of \( \mathcal{A} \) is closed.

Hint: Use descending induction on degrees.

Exercise 2.3. Let \( G \) be a reductive group acting on a vector space \( R \). Lift this action to an action on \( T^* R \). Consider the action of \( \mathbb{C}^\times \) on \( T^* R \) given by \( t.(u,u^*) = (u,t^{-1}u^*) \), and take a character \( \theta : G \to \mathbb{C}^\times \). Let \( f \in \mathbb{C}[T^* R]^{G,\theta} \). Show that every homogeneous component of \( f \) (w.r.t. the grading on \( \mathbb{C}[T^* R] \) induced by the \( \mathbb{C}^\times \)-action) is again in \( \mathbb{C}[T^* R]^{G,\theta} \).

Hint: The actions of \( \mathbb{C}^\times \) and \( G \) commute.

Exercise 2.4. Consider a reductive group \( G \) acting on a vector space \( R \). Show that a quantum comoment map for the action of \( G \) on \( D(R) \) is \( \xi \mapsto \xi_R \).