DAY 2 EXERCISES

1. QUANTIZATION OF ALGEBRAS

Exercise 1.1. Show that the bracket on $\text{gr} \mathcal{A}$ is well-defined and is a Poisson bracket.

Exercise 1.2. Show that the product in $\mathcal{A}/\mathcal{G}$ is well-defined.

2. QUANTIZATION OF SHEAVES

Exercise 2.1 (Exercise 2.1 in the notes). Let $\mathcal{A}$ be a (complete and separated) quantization of $\mathcal{A}$. Show that if $\mathcal{A}$ is Noetherian so is $\mathcal{A}$.

Exercise 2.2. Let $\mathcal{A}$ be a filtered algebra, complete and separated. Assume that $\text{gr} \mathcal{A}$ is Noetherian. Then any left ideal of $\mathcal{A}$ is closed.

Exercise 2.3. Let $G$ be a reductive group acting on a vector space $R$. Lift this action to an action on $T^*R$. Consider the action of $\mathbb{C}^\times$ on $T^*R$ given by $t.(u,u^*) = (u,t^{-1}u^*)$, and take a character $\theta : G \to \mathbb{C}^\times$. Let $f \in \mathbb{C}[T^*R]^G_{\lambda \theta}$. Show that every homogeneous component of $f$ (w.r.t. the grading on $\mathbb{C}[T^*R]$ induced by the $\mathbb{C}^\times$-action) is again in $\mathbb{C}[T^*R]^G_{\lambda \theta}$.

Exercise 2.4. Consider a reductive group $G$ acting on a vector space $R$. Show that a quantum comoment map for the action of $G$ on $D(R)$ is $\xi \mapsto \xi_R$. 