Topics: Commutative rings and their ideals, prime ideals, analyses of ring homomorphisms. Principal ideal domains, Euclidean domains, unique factorization domains. Gaussian integers. Fields of quotients, criteria for irreducibility, the Chinese Remainder Theorem. Modules and module homomorphisms, Structure Theorem for Modules over a PID and its applications, classification of finite-dimensional modules over a polynomial ring in one letter, a module-theoretic description of the Jordan normal form, Cayley-Hamilton Theorem over an arbitrary commutative ring. Chain conditions for modules, Hilbert Basis Theorem, Krull dimension of a commutative ring, Akizuki's characterization of commutative artinian rings, Noetherian (Artinian) induction. Filtration, length, the Jordan-Hölder Theorem. Hilbert-Samuel polynomial, Hilbert polynomial, associated graded ring. Infinite abelian groups, torsion and primary groups, primary decomposition for torsion groups and its uniqueness, divisible groups, Prüfer groups, injective modules, characterization of injective modules over a PID. Algebraic numbers, algebraic integers, the trace map, additive structure of rings of algebraic integers. Invertible ideals and Dedekind domains, Dedekind domains are integrally closed, the Picard group and unique factorization, the norm of an element in a number field, the norm form, the class number of a number field, unit groups. Time permitting, I will also cover the beginnings of algebraic geometry.

There is no single textbook that contains all of the above topics. This year the text will be:


If you need a secondary reference, most of the topics above (but not all) can be found in other general textbooks on algebra. Some of my colleagues at other schools like "Abstract Algebra", 2nd edition, by Dummit and Foote. Others prefer "Algebra", 2nd edition, by Michael Artin, or "Abstract Algebra", 2nd edition, by P.A. Grillet. These are not the only choices, take a look at "Algebra: Chapter 0" by P. Aluffi. There are also several other texts. For some students, a secondary reference may serve as a primary reference, so if you already own one of those texts, see if it works well for you. And if you are averse to reading texts, you don't have to buy a book - my lectures are self-contained.

The theoretical material will be complemented by challenging HW problems, given at regular intervals.

Your final grade will be based on your HW performance.