A Dirac-type operator on a complete Riemannian manifold is of Callias-type if its square is a Schrödinger-type operator with a potential uniformly positive outside of a compact set. We develop the theory of Callias-type operators twisted with Hilbert C*-module bundles and prove an index theorem for such operators. As an application, we derive an obstruction to the existence of complete Riemannian metrics of positive scalar curvature on noncompact spin manifolds in terms of closed submanifolds of codimension-one. In particular, when $N$ is a closed even dimensional spin manifold, we show that if the cylinder $N \times \mathbb{R}$ carries a complete metric of positive scalar curvature, then the (complex) Rosenberg index on $N$ must vanish.