1. Consider the first-order partial differential equation \( u_x^2 + u_y^2 = 9u \).
   (a) Is this equation quasilinear?
   (b) If we impose the initial condition \( u(x, 0) = 1 \), is there a unique solution?
   (c) Obtain the solution(s) satisfying the initial condition in (b).

2. Consider the equation \( u_{xx} - y^2 u_{yy} = y u_y \).
   (a) Determine for what values of \( x, y \) the equation is hyperbolic.
   (b) Where the equation is hyperbolic, find the characteristic curves and use them to find the general solution of the equation.
   (c) Can you impose initial conditions along \( y = 0 \) and find a unique solution?

3. Consider the 2D wave equation \( u_{tt} = \frac{1}{2} (u_{xx} + u_{yy}) \) with initial conditions:

   \[
   u(x, y, 0) = \begin{cases} 
   1 & \text{for } 1 < x^2 + y^2 < 4 \\
   0 & \text{otherwise}
   \end{cases}
   \]

   \( u_t(x, y, 0) = 0 \) for all \( x, y \).

   (a) Find the value of the solution at \( x = 0 = y, t = 1 \).
   (b) Find the value of the solution at \( x = 2 = y, t = 1 \).
   (c) Find the value of the solution at \( x = 1.5, y = 0, t = 0.5 \).

4. Suppose \( B_1 \) is the unit ball centered at the origin in \( \mathbb{R}^3 \), and \( u \in C^2(\overline{B_1}) \) satisfies \( \Delta u = 0 \) in \( B_1 \) and the boundary condition

   \[
   u(x, y, z) = z \quad \text{on } S^2_+ = \text{upper half sphere}
   \]

   \[
   u(x, y, z) = 0 \quad \text{on } S^2_- = \text{lower half sphere}.
   \]

   (a) Find the value of \( u \) at \( (x, y, z) = (0, 0, 0) \).
   (b) Find the maximum value of \( u \) on \( B_1 \).
   (c) Find the sign of \( \frac{\partial u}{\partial z} \) at \( (x, y, z) = (0, 0, 1) \).

5. Consider the equation \( u_t = u_{xx} + 3u_{yy} - x^2 \) for \( 0 < x, y < 1 \) and \( t > 0 \) with homogeneous initial/boundary conditions \( u = 0 \) at \( t = 0 \) and \( u = 0 \) on the boundary of the square \( 0 < x, y < 1 \).

   (a) Show uniqueness of the solution.
   (b) Show that \( u(x, t) \leq 0 \) for \( 0 < x, y < 1 \) and \( t > 0 \).
   (c) What is the sign of \( \frac{\partial u}{\partial x} \) at \( x = 0, y = 1/2 \), and any \( t > 0 \)?