1. Every non-zero homomorphic image of a local ring is local. Prove it.

2. Determine, up to isomorphism, the cyclic modules over the rings in (a) and (b):
   (a) \( \mathbb{Q}[x]/(x^4) \)
   (b) \( \mathbb{Q}[x]/(x^2 - 3x + 2) \)
   (c) Determine all the prime ideals of \( \mathbb{Z}[x]/(x^2 - 3x + 2) \).

3. State and prove sufficient condition(s) on a multiplicative subset \( S \) of \( \mathbb{Z} \) which insure that \( S^{-1}\mathbb{Z} \) is the field of rationals.

4. (a) Let \( G = \mathbb{Z}/(260) \oplus \mathbb{Z}^2 \oplus \mathbb{Q} \). Find a divisible group (injective \( \mathbb{Z} \)-module) \( D \) and an inclusion \( G \rightarrow D \). Justify your statement.
   (b) Find the quotient of \( 7/25 \) by \( n = 15 \) in the divisible group \( \mathbb{Z}(5^\infty) \)
   (c) Explain why \( \mathbb{Z}_{35} \) is not divisible.

5. EXAMPLES. In each case justify your answer by giving a proof.
   (a) Give an example of a commutative ring \( R \) and an element \( x \in R \) which is irreducible but not prime.
   (b) Give an example of a commutative ring \( R \) and an element \( x \in R \) which is prime but not irreducible.
   (c) Give an example of a ring \( R \) and a projective \( R \)-module which is not free.
   (d) Give an example of a ring \( R \) so that \( R \oplus R \cong R \oplus R \) as left \( R \)-modules (i.e. which does not have the invariant dimension property).
   (e) Give an example of a split short exact sequence over the ring \( R = \mathbb{Z}[x] \).
   (f) Give an example of a non-split short exact sequence over the ring \( R = \mathbb{Z}[x] \).