1. Prove that there is a ring isomorphism $\mathbb{R}[x]/(x^3 - 2) \cong \mathbb{R} \times \mathbb{C}$.

2. Let $R$ be a commutative ring. Consider the formal power series ring $R[[x]] = \{ \Sigma_{i=0}^{\infty} a_i x^i \mid a_i \in R \}$.
   (a) Let $f = \Sigma_{i=0}^{\infty} a_i x^i \in R[[x]]$. Prove that $f$ is invertible if and only if $a_0$ is a unit in $R$.
   (b) Let $g = 1 - x + x^2$. Find $g^{-1} \in R[[x]]$.

3. Determine, up to isomorphism, the cyclic modules over the rings in (a) and (b):
   (a) $\mathbb{Q}[x]/(x^4)$
   (b) $\mathbb{Q}[x]/(x^2 - 3x + 2)$
   (c) Determine all the prime ideals of $\mathbb{Z}[x]/(x^2 - 3x + 2)$.

4. Let $R = \mathbb{Z}$. Consider the $R$-module $M = \mathbb{Z}/(24)$.
   (a) Find two submodules $M_1, M_2$ of $M$ such that $M = M_1 + M_2$ and $M_1 \cap M_2 = 0$, i.e. so that $M$ is internal direct sum of its submodules $M_1, M_2$. Prove your statement.
   (b) Describe cyclic module decomposition of each of the modules $M_1$ and $M_2$.

5. Consider the following commutative diagram and short exact sequences of modules:
\[
\begin{array}{ccc}
0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\
 & & f_1 & & f_2 & & f_3 & & \\
0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & 0 \\
\end{array}
\]
Prove the following statements:
   (a) If $f_2$ is injective, so is $f_1$.
   (b) If $f_3$ is isomorphism then $\text{Ker}f_1 \cong \text{Ker}f_2$ and $\text{Coker}f_1 \cong \text{Coker}f_2$, where $\text{Ker}$ stands for the kernel of the homomorphism and $\text{Coker}$ stands for the cokernel.

6. (a) Let $G = \mathbb{Z}/(300) \oplus \mathbb{Z}^2$. Find a divisible group (injective $\mathbb{Z}$-module) $D$ and an inclusion $G \rightarrow D$. Justify your statement.
   (b) Is $5/27$ divisible by $39$ in $\mathbb{Z}(3^\infty)$? If yes - find the quotient, if not - explain.