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Dissertation Defense Abstract

Convex Polytopes and Tilings with Few Flag Orbits

The amount of symmetry possessed by a convex polytope, or a tiling by convex polytopes, is reflected by the number of orbits of its flags under the action of the Euclidean isometries preserving the polytope. The polytopes with the fewest such orbits are the "most symmetric". The convex polytopes with only one flag orbit have been classified since the work of Schläfli in the 19th century. In this dissertation, a classification is given for convex polytopes with up to three flag orbits. A description of the polygons in the plane with any number of flag orbits is also given. The resulting flag orbits from various standard operations used to construct new polytopes from others are also discussed. Finally, it is proven that for any k, the polytopes with k flag orbits occur in only finitely many dimensions, and that a certain class of d-dimensional polytopes (including most uniform polytopes) is either regular or has at least d-1 flag orbits.