

MATH 4020 – Research Capstone – Syllabus A for Fall 2015

Time: Meeting in Sequence 3: M W Thu 10:30-11:35 at 408 Ell (off main quad).
Prof. Iarrobino, 526 NI x 5524. e-mail a.iarrobino@neu.edu (best way to reach me besides dropping by office).

Math 4020 satisfies NU Core Capstone, NU Core Writing Intensive in major.

Text: A (Terse) Introduction to Linear Algebra, Yitzkhak Katznelson and Yonatan Katznelson, Student Mathematical Library #44 (2008), American Mathematical Society, ISBN-978-0-8218-4419-9. (at bookstore)

Reference, optional reading: Mathematical Connections: a Capstone Course: John Conway.
American Mathematical Society, (2010). ISBN 978-0-8218-4979-8
(This gives an overview of some topics in math of possible interest).

These two books are quite inexpensive and Conway may be shared, I will have a copy of Conway available for loan, both are ordered at the NU Bookstore).

Advised Prerequisites: Linear Algebra Math 2331, Group Theory Math 3175, and either Math 3150 Real Analysis or Math 4565 Topology or permission of instructor.

Other intermediate/advanced math course(s) such as Math 4525 Applied Analysis, Math 4571 Advanced Linear algebra, Math 3527 Number Theory, Math 3560 Geometry, 4565 Topology, 3541 Chaotic Dynamical Systems, Math 3533 Combinatorial mathematics, Math 4545 Fourier Series and PDE's may lead to a topic for your research.

For Who: The course is designed primarily for junior and senior Mathematics majors, who wish an introduction to mathematical research, or to develop more deeply some direction of research already begun.¹ In comparison with Math 5131 (Introduction to Mathematical Methods and Modeling – taught in fall) Math 4025 Applied Math capstone (taught in Spring), Math 4020 has more emphasis on writing an original student research project, that may be applied, pure, or some combination. Plan is to give sufficient support as needed so each student can complete a research paper (possibly in collaboration). Most students will be matched with a faculty consultant, and the availability of a suitable math faculty consultant may affect the possible choices.

Summary: The components are

- I. Research project (80% of grade).

¹ Some students may choose to continue their work in a subsequent semester via an Independent Study with a faculty member who accepts to supervise such. This course may also be followed by Math 4970 Junior-Senior Honors Project, leading to graduating with "Honors in Mathematics."

- II. Reflection on A. an ethical issues and B. on the process of research you engaged in (10%).
- III. Class participation including class notebook, presentation of topics in advanced linear algebra, of topics related to project. (up to 10%).
(Students to have 5% flex to add weight to favor one of I,II,III).

I. RESEARCH component 80% of course grade (agree with instructor)

Each student will be assigned a research project agreed upon by the student and approved by the instructor and depending upon the interests of the individual student. Some topics may be worked on jointly by several students. The instructor will endeavor to match student interests with an appropriate faculty member as a consultant. Your topic may be in any area of mathematics or mathematics application provided there is a suitable consultant available. See the Mathematics Department Undergraduate Research Page ² and links there for some possible topics.

The topic will require an element of student initiative beyond learning about or reporting on the work of other mathematicians. Research as defined for this course does **not** expect work that is new to mathematics, but **does expect** work under some guidance/supervision that is new to the student and discovered by the student's exploration on his/her own.

The student should maintain a research journal in which each page is dated and which contains an accurate account of the student's progress. The student should be prepared at any point to answer questions about the status of the project, and upon a one-week notice, to give a talk to the class.

Students will be expected to participate in weekly discussions of their own work and those of others., and to present topics of interest to the class. Students will be expected to keep a separate notebook summarizing these discussions, and questions, comments they have. Credit will be given for helpful suggestions, feedback. (10% of course grade class discussion and commentary notebook, and presentations before the final presentation).

Students are permitted to consult with each other, other faculty and any and all research sources. However, all work that is not the student's must be cited in he research notebook, including casual conversations with fellow students if that is a source of insight. Contributions of others will be appropriately cited in the References of the undergraduate research report/paper.

At least twice during the semester the student will be required to present a preliminary report detailing the current state of the project and future directions.

² <http://www.northeastern.edu/cos/mathematics/undergraduate-program/undergraduate-research/>

Schedule of work on research and check points:

End of week 1	Rough idea of topic.
End of week 2	Written draft and description of topic and projected goal; rough idea for carrying out project; description of mathematics needed, identify a support faculty or grad student. (5% of course grade)
Weeks 3-6	Basic work;
Week 6:	Written progress report; preliminary presentations; history of the problem and its significance including definitions, examples and applications. (5% of grade)
Weeks 7-9	Advanced work; Significant progress should be made Meet weekly with instructor outside of class for 15-30min).
Week 10	Penultimate drafts due; preliminary presentations. (5% of course grade)
Week 11	Clean up, clear up, rewrite.
Next to last week	Final papers due.
Last two weeks	Final presentations (approximately 15-20 minutes) (5% of course grade)

More on the Paper

There is no fixed length but six to twelve pages for the main body (parts 4-7 below) is not an unreasonable amount. The final report should be prepared with LaTeX³ be spell checked, be grammatically correct and include proper bibliographic citations. All work that is not the student's must be cited.

The paper should have the following.

- 1) Title page (Project title, your name,, department, date, faculty contact, advisor, if not instructor).
- 2) Abstract (Short description; no more than 300 words).
- 3) Table of Contents
- 4) Introduction (Describe problem. Describe your results. Explain your approach. List the major results.)
- 5) Definitions and notations. (Emphasize any definitions you came up with yourself. Emphasize any unordinary notations.)
- 6) Details (Don't just list theorems and proofs but included descriptive exposition around the theorems and proofs.).
- 7) Conclusions, discussion and possible future research.
- 8) Index of Notations (if needed)
- 9) References (Include published works and discussions you had with other student or faculty)
- 10) Appendices (if needed =Include any program simulations run).

³ Or alternate format/ word program agreed in advance with the instructor.

(The paper itself is 60% of the course grade).

II. A. Reflective essay A: 2-3 page discussion of an ethical issue pertaining to either mathematics research, or the use of mathematics by a practitioner. Paper should include or propose an exercise or case study for class.

B. Reflective Essay B: Required 2-5 page reflection on the process of research you engaged, in and any observations of yourself as a mathematician/ student. This may include suggestions about the course, and about how you might undertake another project differently.

(II is 10% of course grade).

III. Presentation-Class Notebook: Linear Algebra and Applications component:

This is intended as a support to the research project, and topics may be adjusted according to student interest or need.

Source: Text: KK Chapter 1. Vector Spaces, 2. Linear operators and Matrices, 3. Duality of vector spaces.

5. Invariant Subspaces: characteristic polynomial, invariant subspaces, minimal polynomial

7. Structure Theorems: reducing subspaces, semisimple systems, nilpotent operators, Jordan canonical form, cyclic decomposition.

Supplementary source:

Conway: Chapter 5. Matrices and Topology: This is closer to real analysis.

Chapter 6 Modules; This optional source gives an opportunity to view the Structure Theorems of KK from a module viewpoint.

III: Presentations and class notebook are 10% of course grade.

Plan for first two weeks of course.

First week: Day one: Plan for course, interests of students.

Class 1-3:

Review / extension of linear algebra as needed with students presenting topics from K. [Goal, practice presenting, review of linear algebra, Jordan normal form]. Students choose initial readings related to a potential topic.

Initial individual meeting with instructor during first 2 weeks outside of class at least once 15-30 minutes.

Possible visits and short presentations by faculty willing to be consultants.

Second week: Continued presentations. Some first presentations from readings related to a possible project.

End of second week: Draft of proposed topic due (see above).

EXAMPLES of possible research topics and potential project advisors/consultants.⁴ You may also seek out a faculty consultant.

Examples ...

- A. When do two nilpotent matrices commute. This involves knowledge of the Jordan normal form (see part III) and some exploration of combinatorics.
- B. Behavior of the characteristic polynomial of a matrix and its eigenvalues under a deformation of the matrix (add a perturbation)
- C. The Exercises in Conway: Chapters 5 (Matrices and Topology), Chapter 6 (Modules) are not in themselves a research topic, but might very well lead to one.
- D. Infinite measure-preserving random walks, tilings, does 2-mixing imply 3-mixing (project advisor Prof. Eigen),
- E. Modeling time series as sparse Probability Suffix Trees, using these for statistical inference, anagram ambiguity (Prof. Malioutov).
- F. Maxwell point charges (Prof. Gaffney)
- G. Topology of Singular spaces (Prof. Massey)

Appendix 1: Tips on writing mathematical papers:

How to write mathematics, corrected edition, by Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer, and Jean A. Dieudonné American Mathematical Society, Providence RI 1981, ISBN 0-8218-0055-8. A reprint of four papers.

Ten simple rules for mathematic writing: Dimitri Bertsekas, MIT 2002

http://www.mit.edu/~dimitrib/Ten_Rules.pdf

Mathematical Writing Donald E. Knuth, Tracy L. Larrabee, and Paul M. Roberts (Washington, D.C.: Mathematical Association of America, 1989), ii+115pp. ISBN 0-88385-063-X (see <http://tex.oria.fr/typographie/mathwriting.pdf>).

Guide to writing Mathematics: Kevin Lee

<http://www.cs.ucdavis.edu/~amenta/w10/writingman.pdf>

Mathematical writing: a brief guide: Mark Tomforde.

<http://www.math.uh.edu/~tomforde/MathWriting.pdf>

This is really excellent, brief and mentions TeX, LaTeX, and has references.

LateX guide: Prof. Richard Porter (Northeastern University)

Some Remarks on Writing Mathematical Proofs, John Lee (U. Washington)

<http://www.math.washington.edu/~lee/Writing/writing-proofs.pdf>

This is also excellent on its topic.

Writing a Math Phase Two Paper: Steve Kleiman (MIT)

<http://www-math.mit.edu/phase2/UJM/vol1/KLEIMA~1.PDF>

This is also excellent, and quite detailed, has references.

⁴ For further detail on some of these possible topics see the Math Dept page <http://www.northeastern.edu/cos/mathematics/undergraduate-program/undergraduate-research/research-topics-problems/>

Here is another resource with links on writing mathematics from Math Stack exchange:

<http://math.stackexchange.com/questions/14189/what-are-or-where-can-i-find-style-guidelines-for-writing-math>

Terence Tao blog: <https://terrytao.wordpress.com/advice-on-writing-papers/>

Appendix 2: Undergraduate Mathematics Research resources:

Mathematics Department undergraduate research pages:

<http://www.northeastern.edu/cos/mathematics/undergraduate-program/undergraduate-research/>

Potential Math 4020 Topics/consultants in Math Department

<http://www.northeastern.edu/cos/mathematics/undergraduate-program/undergraduate-research/research-topics-problems/>

Conferences:

<http://www.maa.org/programs/students/undergraduate-research/opportunities-to-present>

<http://www.maa.org/programs/maa-grants/RUMC>

Hudson River Undergraduate Mathematics Conference (April)

<https://spock.skidmore.edu:8002/apex/f?p=114:1:5176040639552163::::hrumc%2Fapex%2Ff>

Summer REU's: before graduating, so juniors or below: deadlines tend to be in January of the year in question:

<http://www.ams.org/programs/students/undergrad/emp-reu>

Terence Tao blog: "Career Advice"

<https://terrytao.wordpress.com/career-advice/>