Robust Multifrequency Inversion in Terahertz Tomography
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Motivation
- **Problem**: The developed joint multifrequency inversion technique outperforms single-frequency inversion, but requires complete and accurate spectral information.
- **Goal**: Robust reconstruction approaches which do not assume exact prior information of spectra.

Research to reality
This work provides a general framework for robust joint image formation using multiple frequencies for improved reconstruction and subsequent recognition. The algorithms are applicable to Terahertz (THz) pulsed imaging which has attracted much attention for its ability to reveal unique spectral characteristics of chemicals in THz range and thus to fingerprint explosives.

Background
Object function: 
\[ \alpha(r, f) = \bar{u}(r) - u_0(r) \]
Measurement: 
\[ u_s(r) = u(r) - u_0(r) \]
Fourier Diffraction Theorem \cite{[1]} relates scattering with object spectrum under the Born approximation \[ u(r) \approx u_0(r) \]

Model: \( \{ f_m \} \) in THz radiation spectrum
\[ y_m = \Psi_m x_m, \quad m = 1, \ldots, M \]
\( y_m \in \mathbb{C}^{N} \) Fourier transform (FT) of THz field measurement
\( x_m \in \mathbb{C}^{N} \) Estimate of object function at \( f_m \)
\( \Psi_m \) Nonuniform FT operator with fast approximationNUFFT\cite{[2]} \( T_m \approx \Psi_m \)

Joint inversion with \( \mathcal{E} \)
\[ \text{JMP: } \min_{\{ x \}} \| y - \mathcal{T} \mathcal{H} x \|_2^2 + \alpha_1^2 \| D^2 x \|_W^2 + \varphi(s, \gamma) \]

Case 1. With partial \( \mathcal{E} \)

RMJP1: \[ \min_{\{ x \}} \| y - \mathcal{T} \|_2^2 + \alpha_1^2 \| D^2 x \|_W^2 + \varphi(s, \gamma) \]

Case 2. With uncertain \( \mathcal{E} \)

- Uncertainty modelling: \( \varepsilon_j - \Delta_j \leq s_j \leq \varepsilon_j + \Delta_j \)
- Inverse with worst-case prior: \( \mathcal{T} = (e_1, \ldots, e_N) \otimes I_W \)

RMJP2: \[ \min_{\{ x \}} \| y - \mathcal{T} \mathcal{H} x \|_2^2 + \varphi(s, \gamma) \]

NP-hard solving \( \psi \): possible solution \( \psi_j(s, \gamma) \) at \( \mathcal{H} \), \( j = 1, \ldots, 2^J \)

Smoothing approximations: \( \psi_j(s, \gamma) \leq \lambda^{-1} \log \left( \sum_j \exp(\lambda_j s_j) \right) \)

References

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