Euclidean Structure Recovery from Motion in Perspective Image Sequences via Hankel Rank Minimization

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Abstract

We consider the problem of **recovering 3D Euclidean structure from multi-frame point correspondence data in image sequences under perspective projection.**

Here we introduce a new constraint that implicitly exploits the *temporal ordering* of the frames, leading to a provably correct algorithm to find Euclidean structure (up to a single scaling factor) without the need to estimate the Fundamental matrices or assume a camera motion model.

The proposed approach does not require an accurate calibration of the camera.

Relevance

Dynamics-Based 3D geometry reconstruction enables discrimination of anomalous shapes and automatic mapping for autonomous navigation.

Existing approaches:

- •Find projective geometry by iteratively minimizing the rank of the measurement matrix up to a projective transformation
- •Discard temporal information
- •Try to solve a challenging nonlinear optimization problem, which is sensitive to initialization, and convergence is not guaranteed.

Our Approach:

- •Convex optimization based solution
- • Avoids estimation of epipolar geometry and fundamental matrix
- • Uses the temporal ordering of the data
- • Finds accurate 3D structure up to a "**single scaling**" factor

Technical Approach

BACKGROUND:

The Hankel Matrix of a vectors sequence $\{\mathbf{y}_k\}_{k=1}^{n+l-1}$ is defined as :

$$\mathbf{H}_{\mathbf{y},n,l} \doteq \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_l \\ \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_{l+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_n & \mathbf{y}_{n+1} & \cdots & \mathbf{y}_{l+n-1} \end{bmatrix}$$

Embeds temporal information (reordering of the sequence changes the rank of the matrix)

MAIN RESULT:

Let $\mathbf{P}_{ij} = (X_{ij}, Y_{ij}, Z_{ij})^T,\ j = 1,2,3$ be the 3D Cartesian coordinates of 3 points on a moving object, with 2D projections $(u_{i,j},\ v_{i,j})$. Then, the **correct** depths Z_{ij} minimize the rank of the Hankel matrix $\mathbf{H} = [\mathbf{H}_y^{13}\ \mathbf{H}_y^{23}]$

where $$\mathbf{y}_k^{ij} = \begin{bmatrix} \frac{1}{f}(Z_{ki}^* u_{ki} - Z_{kj}^* u_{kj}) \\ \frac{1}{\alpha f}(Z_{ki}^* v_{ki} - Z_{kj}^* v_{kj}) \\ Z_{ki}^* - Z_{kj}^* \end{bmatrix}$$

Dynamics encapsulate 3D Geometry!

ALGORITHM:

$$\min_{\{Z_{k1}^*, Z_{k2}^*, Z_{k3}^*\}_{k=1,\ldots,F}} \operatorname{rank}\left([\mathbf{H}_{\mathbf{y}^{13}} \quad \mathbf{H}_{\mathbf{y}^{23}}]\right)$$ subject to a linear + a rank constraint

NP-HARD! BUT **Semi Definite Embedding:**

$$\begin{aligned} &\textit{minimize} \ \ \mathbf{Rank}(\mathbf{X}) \\ &\textit{subject to} \ \ \mathbf{X} \in C \end{aligned}$$

CONVEX RELAXATION

$$\begin{aligned} &\textit{minimize} \ \ \tfrac{1}{2}\mathbf{Rank}(\mathbf{diag}(\mathbf{Y},\mathbf{Z})) \\ &\textit{subject to} \ \ \begin{bmatrix} \mathbf{Y} & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Z} \end{bmatrix} \geq 0, \ \ \mathbf{X} \in C \end{aligned}$$

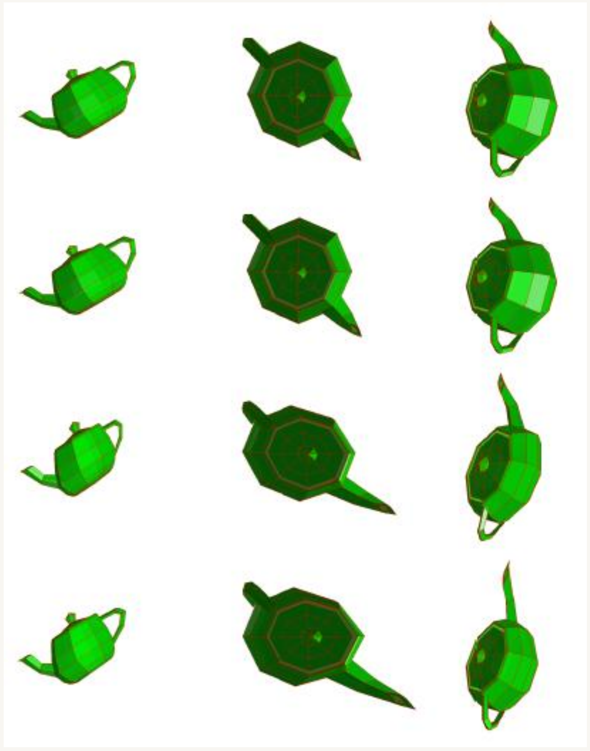
Can be solved by LMIrank

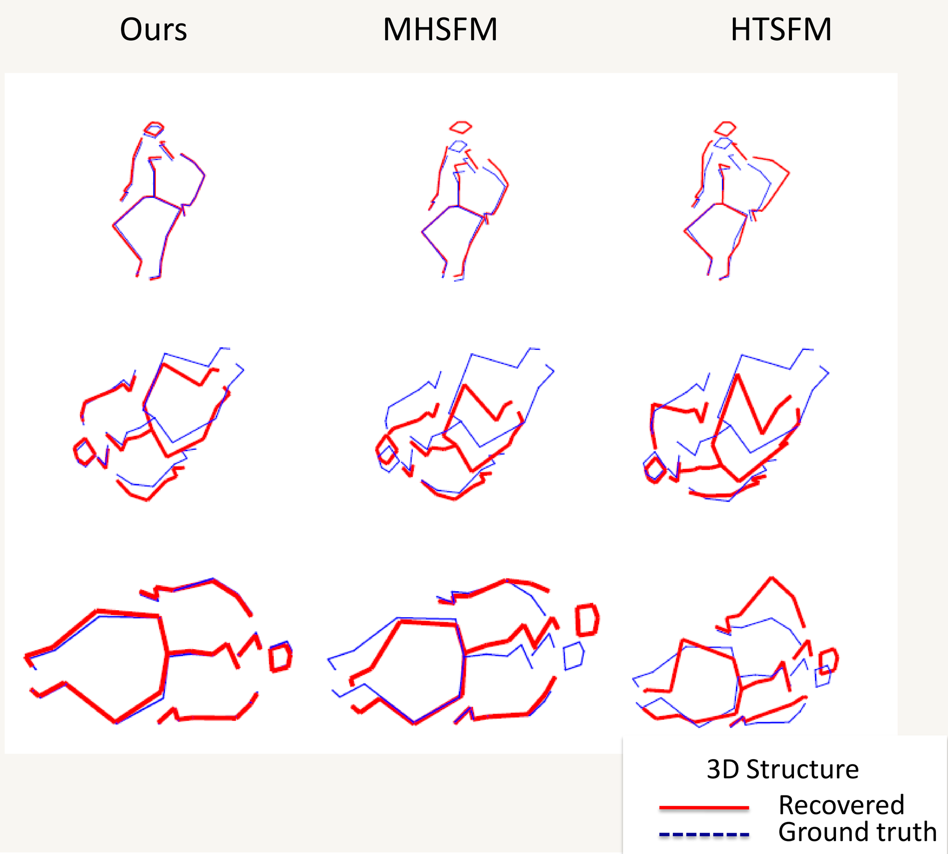
To construct the rest of the object:

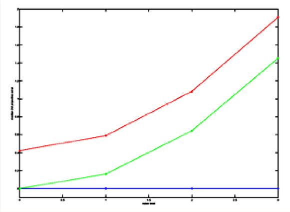
$$\min_{\mathbf{s}, Z_{k5}} \|\mathbf{W} \cdot \mathbf{s} - \mathbf{P}_5\|$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{P}_{11} & \ldots & \mathbf{P}_{14} \\ \vdots & \ldots & \vdots \\ \mathbf{P}_{F1} & \ldots & \mathbf{P}_{F4} \end{bmatrix}; \ \mathbf{P}_5 \doteq \left[\tfrac{1}{f} Z_{15} u_{15} \ \ \tfrac{1}{f\alpha} Z_{15} v_{15} \ \ Z_{15} \ \ \ldots \ \ \tfrac{1}{f} Z_{F5} u_{F5} \ \ \tfrac{1}{f\alpha} Z_{F5} v_{F5} \ \ Z_{F5}\right]^T$$

RESULTS:

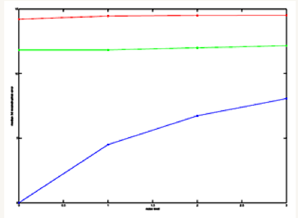






2D error plot

Umbrella Object



3D error plot (Blue line is our algorithm)

Human Chair Object

Table 1. 3D and 2D re-projection median error.

Data Set	HankelSFM		MHSFM		HTSFM	
	3D (mm.)	2D (pixels^2)	3D (mm.)	2D (pixels^2)	3D (mm.)	2D (pixels^2)
Teapot (R)	4.89e-1	0	1.34e+1	3.5e+1	1.34e+1	1.2e-7
Teapot(RT)	1.61e-4	0	3.00e+1	1.0e+1	3.20e+1	2.5e-7
Umbrella	3.50e+1	0	8.22e+1	0.6176	8.32e+1	0.0136
Human	4.10e+1	0	1.37e+2	2.3091	1.51e+2	0.2713

Accomplishments Through Current Year

- •Algorithm Development
- •Efficient Implementation of the method
- •Early testing of the algorithm and comparison with existing methods

Future Work

Research is currently underway to extend these results to articulated and non-rigid objects.

Opportunities for Transition to Customer

3D Reconstruction from video allows us to recover Euclidean shape from sequences of images. The recovered models can be used to analyze body shape and detect anomalous or suspicious bulks.

Additionally, the proposed method can be used for marker-less motion capture for medical and sport applications and for 3D mapping for autonomous navigation.

Patent Submissions

Not Available

Publications Acknowledging DHS Support

•Ayazoglu M., Sznaier M., and Camps O.: Euclidean Structure Recovery from Motion in Perspective Image Sequences via Hankel Rank Minimization. IEEE ECCV, (2010)

Other References

•Mahamud, S., Hebert, M.: Iterative projective reconstruction from multiple views. IEEE CVPR, (2000)

•Hung, Y., Tang, W.: Projective reconstruction from multiple views with minimization of 2d reprojection error. IJCV, (2006)

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