I. Reconstructions

An 81 × 81 phantom consisting of 3 explosives and air as background was generated with spectral prior given in [5][6]. We simulated THz incident fields at 19 projection angles, and collected complex amplitude of the scattering at 4 frequencies for reconstruction. Each method was evaluated under various levels of Gaussian noise. Every pixel in the reconstructed fields was classified using rule as follows: For Methods I and II, the class is chosen to minimize the Euclidean distance from the reconstructed susceptibilities at different frequencies to the spectral priors of the different explosives. For Method III, the most likely component is assigned to that pixel.

II. Performance comparisons

Spatial information may not be well extracted at certain frequencies in Method I due to factors such as lower resolution (i.e., longer wavelength) and feature ambiguity. Method II, the joint multifrequency approach, improves the estimation of the boundary field and thus enhances the accuracy of the reconstruction and the subsequent recognition. Method III achieves better reconstruction and recognition by imposing spectral prior into the inverse process and consequently forcing spatial consistency during the reconstruction.

References


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Background

I. Modality: transmission tomography

Object field: \( \alpha(r, f) = \hat{n}(r, f)^2 - 1 \)

Measurements: \( u_i(r) = u(r) - u_0(r) \)

- Scattered field in the form of Green’s function
- Written in Fourier transform terms under the first Born Approximation \( u_i(r) \approx u_0(r) \)

\( U_i(\omega) = G(\omega) \{ O(\omega) \ast U_0(\omega) \} = G(\omega)2\pi\Omega(\omega - k) \)

\[ u_i(r) = \frac{1}{i\omega_0} \int g(r - r')\alpha(r') u(r') \, dr' \]

\[ \rho(r) \mu(\omega_i, k_0) \mathcal{F}_2 \{ u_i(r) \} (\omega) \]

\[ u(r) \rightarrow \mathcal{F}_i \{ u_i(r) \} \]

\[ F_{2D} \{ \alpha(r, f) \} \]

Fouier Diffraction Theorem relates scattering with object spectrum

II. Nonuniform FFT

\[ T: \text{ Fast approximation for the NUFT}^2 \]

step 1. Point-wise scaling
step 2. Oversampled FFT
step 3. Min-max optimized Kaiser-Bessel interpolation using small local neighborhoods

Motivation

Much of the recent interest in terahertz (THz) imaging stems from its ability to reveal unique spectral characteristics of chemicals in THz range and thus to fingerprint explosives. Short-pulse THz sources provide broadband excitation, but most inversion techniques as diffraction tomography work construct images for single frequencies. In this work, we explore alternatives for joint image formation using multiple frequencies for enhanced explosives detection.

Methods

I. Reconstruct frequency by frequency

Reconstruct object field \( x_m \in \mathbb{C}^{N^2} \) and boundary field \( s_m \in \mathbb{R}^{N^2} \) at each frequency \( f_m \), \( m = 1 \ldots M \):

\[ \| y_m - T_m x_m \|_2 + \alpha_2 \| D x_m \|_2 + \beta_2 \| D s_m \|_2 + \frac{1}{2\beta_2} \| s_m \|_2 \]

- Data-fidelity term in frequency domain
- Smoothness penalty term in spatial domain, where \( D \) is a derivative operator
- Spatially varying weighting: \( W_s = \text{diag}(1 - |s_m|^2) \)
- Alternating coordinate minimization
- Speed up with \( T_m = \Psi_m \)

II. Joint multifrequency and spatial prior

- Boundary field \( s \) is invariant across frequencies
- Joint multifrequency to reconstruct \((\mathbf{x}, \mathbf{s})\):

\[ \mathbf{g} = [x_1 \ldots x_M]^T \]

\[ \| y - T \mathbf{g} \|_2^2 + \alpha_2 \| D \mathbf{g} \|_2^2 + \beta_2 \| D \mathbf{s} \|_2^2 + \frac{1}{2\beta_2} \| \mathbf{s} \|_2^2 \]

where \( T = \text{diag}(T_m) \), matrix with \( \sim \) stands for kronecker product of this matrix with \( I, \dim(I) = M \)

III. Combine spectral priors

- Known \( J \) components with spectral prior \( \nu \in \mathbb{C}^{M \times J} \)

\[ \nu = \nu \mathcal{H}_4(\mathbf{u}) \]

Relation between component concentration fields and object fields

- Linear transform \( \mathcal{H}_4: \mathbf{g} \rightarrow \mathbf{H} \nu \)

- Joint multifrequency to reconstruct \((\mathbf{u}, \mathbf{s})\):

\[ \nu = [\nu_1 \ldots \nu_J]^T \quad \nu_i \in \mathbb{R}^{N^2}, \quad |\nu_i| \leq 1 \]

\[ \| y - T H \nu \|_2^2 + \alpha_2 \| D \nu \|_2^2 + \beta_2 \| D \mathbf{s} \|_2^2 + \frac{1}{2\beta_2} \| \mathbf{s} \|_2^2 \]

where matrix with \( \sim \) stands for kronecker product of this matrix with \( I, \dim(I) = J \)