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Variability, Covariation, and Invariance With Respect to Coordinate Systems in Motor Control: Reply to Smeets and Louw (2007)

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In their comment on the tolerance-noise covariation (TNC) method for decomposing variability by H. Müller and D. Sternad (2003, 2004b), J. B. J. Smeets and S. Louw (2007) show that covariation (*C*), as defined within the TNC method, is not invariant with respect to coordinate transformations and contend that it is, therefore, meaningless. Although the observation is correct, their interpretation is misleading in the following ways: (a) They equate covariation *C* with the known statistical quantity covariance and noise (*N*) with standard deviations. The two quantities *C* and *N* are conceptually different statistical measures. (b) Dependency on the reference frame is not only a feature of *C* but of all 3 components. However, such dependency is ubiquitous in motor control. (c) As the frame of reference in biological systems is poorly understood, the TNC method may afford evaluation of different coordinates for control.

Keywords: variability, covariation, coordinate system, motor control

Variability in movement performance has served as an interesting window into basic mechanisms of control and coordination. Many empirical reports have attested that the magnitude and structure of variability in performance changes with practice, although fluctuations never reach zero, not even in highly skilled performers. To better understand the underlying processes that give rise to these changes in the course of learning, we have developed a method that decomposes the observed variability into three conceptually distinct components (Müller and Sternad, 2003, 2004b). Essentially, the method parses the changes of overall variability in performance into three components: tolerance (*T*), noise (*N*), and covariation (*C*). The objective of this decomposition is to reveal changes not only in the magnitude but also in the structure of variability. For example, although a reduction in performance variability has been commonly equated with a decrease in (stochastic) noise, a reduction of overall variability may also be brought about by compensatory relations between the relevant variables. This change is quantified by covariation. Further, variability that arises as a result of exploration of the workspace in initial phases of learning can be quantified and separated from changes in overall dispersion. These three components are quantified by tolerance and noise, respectively. The method as briefly reviewed below was applied in the two throwing actions of skittles and darts as well as in a pointing task (Müller, 2001; Müller & Loosch, 1999; Müller & Sternad, 2003, 2004a, 2004b, 2007).

Inspired by this work, Smeets and Louw (2007) applied the tolerance-noise-covariation (TNC) decomposition to a dart-throwing action simplified to two dimensions. By describing the action in two equivalent coordinate systems (Cartesian and polar) Smeets and Louw show that the contribution of covariation depends on the coordinate system.[1] They conclude that this dependency on transformations of the coordinate system renders the method ambiguous and of little use. Specifically, covariation as a descriptor of changes over the process of learning should be discarded from the analysis.

We would like to respond to their comment in three steps: (a) Although their calculations are correct, their results are interpreted erroneously. Specifically, the quantities covariation *C* and noise *N* are confused with covariance and variance, respectively. (b) Illustrating the basic steps of the TNC calculations in a simple simulation, we show why the method is indeed dependent on the coordinate system, even more so than Smeets and Louw point out. However, such universality was never claimed and is similarly absent in common statistical measures and other analysis methods used in motor control. (c) The dependence on the coordinate system is a positive and valuable feature, as the TNC method

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[1] Note that the polar coordinate description of Smeets and Louw is not an alternative reference frame for the release coordinates but rather an alternative description of the velocity vector in terms of its magnitude and direction.

describes aspects of control. As such, the method may be used to evaluate candidate coordinate systems hypothesized for control.

Before elaborating these three points, we would like to clarify the basic assumptions and conceptual steps of the TNC method again. The starting point for application of the TNC method is description of the movement of interest on at least two levels: in terms of execution variables ($e_1, e_2, \ldots, e_n$) and in terms of result variables ($r_1, r_2, \ldots, r_m$), where n and m are the number of variables, respectively. Both levels of description are linked through a function f:

$$r_i = (r_{i,1}, r_{i,2}, \ldots, r_{i,m}) = f(e_{i,1}, e_{i,2}, \ldots, e_{i,n}), \qquad (1)$$

where i denotes the ith repetition. Equivalently, the two sets of variables may be considered to define an *execution space* and a *result space*, respectively, and the function f defines the mapping of execution onto results. A simple example is the two-dimensional dart-throwing task of Smeets and Louw, in which the two ($n = 2$) execution variables e_1 and e_2 are the release velocities in x and y directions, V_x and V_y. They completely determine the result. The result r is quantified by the single variable distance to target D ($m = 1$). The function f can be any relation—linear or nonlinear, dynamic or static—that unambiguously maps each single execution into a result. Importantly, the relation between execution and result can be many to one, $n > m$, such that more than one combination of execution variables yields the same result. In that case, the task has a *solution manifold*, containing all "solutions" to the task that have the same result: for example, zero distance or error, $D = 0$. Choosing variables for execution and result and their functional relation f defines the task.

Having selected execution and result variables, the TNC method is applied by comparing two sets (blocks) of data: B_1 and B_2. These data sets can be taken from either two different stages in a learning process or two different experimental manipulations or subjects. The difference in performance between the average results D of the two data sets is obtained by the algebraic difference between the result measures: $\Delta D = D(B_2) - D(B_1)$. ΔD can now be decomposed into three components: ΔT, ΔN, and ΔC, where the sum of the three components exactly equals ΔD (for details see Müller & Sternad, 2004b):

$$\Delta D = \Delta T + \Delta N + \Delta C. \qquad (2)$$

The component ΔT accounts for changes in D from B_1 to B_2 that are due to changes of location in execution space leading to better solutions. ΔT also captures changes in the result when the new location is less sensitive, that is, more tolerant, to variations or perturbations in the execution variables. ΔN quantifies changes in the dispersion of execution variables, and ΔC represents a systematic coupling between execution variables that improves performance (D). Note that all contributions are evaluated by their effect on the result variable D, not in terms of changes of (or relations between) the execution variables. To make this distinction clear, we emphasize that the last component is called *covariation* (ΔC), not covariance, as it quantifies systematic coupling between execution variables that is measured in the units of the result variable. The relation that gives rise to the covariation component ΔC of the difference in results may involve two or more execution variables and may be nonlinear. In contrast, *covariance* is the common statistical measure of the linear association between two variables. It may be used to compute correlation, a dimensionless measure that quantifies the goodness of fit of a linear relation between two execution variables. Similarly, the effect of random execution variability is also quantified by its contribution to the difference in results and is measured by ΔN in the units of the result space. It is not identical to dispersion of the execution variable(s), which may be measured in the usual way by variance or its square root (standard deviation).

Covariation and Noise Are Measured in Result Space

In their interpretation of their simulation results, Smeets and Louw confuse the covariation measure ΔC with covariance among execution variables and similarly noise (ΔN) with the variances or standard deviations of the execution variables. Hence, some of their statements are misleading: For example, Smeets and Louw claim that in a learning process that does not involve changes in covariance of the execution variables, ΔN and ΔC should also remain invariant; that is not the case.

This somewhat subtle but conceptually important point is illustrated in our simulation using the dart-throwing example by Smeets and Louw with the same functional relation for f. Three data sets (B) of 100 trials each were created, and execution was described by the Cartesian release velocities V_x and V_y (see Table 1). Identical to Smeets and Louw's example, the data show improvements in D from B_1 to B_2 to B_3, while the standard deviations of the execution variables V_x and V_y were kept invariant across the blocks. In contrast to Smeets and Louw's simulations, a fixed level of covariance between the execution variables was included that also remained invariant across blocks. Applying the TNC decomposition to these data, performance improvements measured by ΔD are primarily obtained by tolerance ΔT, but there is also a

Table 1
Results of a Dart Throwing Simulation With Three Data Blocks of 100 Trials Each

$Block_i$	V_x (m/s)	V_y (m/s)	SD_x (m/s)	SD_y (m/s)	*cov*	D (m)	ΔC_i (m)
B_1	1.50	6.53	0.08	0.35	.0064	0.567	−0.059
B_2	3.13	3.13	0.08	0.35	.0064	0.197	−0.010
B_3	6.53	1.55	0.08	0.35	.0064	0.090	−0.001

Note. The means and standard deviations of the execution variables V_x and V_y were chosen to be the same as in the data of Smeets and Louw, but the covariances were nonzero and unchanged across blocks. Results illustrate how covariance (*cov*) can be different from covariation (ΔC_i). D denotes the result variable distance to target. i denotes the ith repetition.

nonnegligible contribution of covariation ΔC, especially from B_1 to B_2 (see Table 2). In addition, there is a nonnegligible contribution of noise ΔN. The dispersion in the individual execution variables, measured by their standard deviations, is constant and its changes are (by assumption) exactly zero. However, the noise contribution ΔN, computed in terms of its effect on the result variable, decreased across blocks. The effects of these distinctions are illustrated in Table 2.

These simulations demonstrate that Smeets and Louw's conceptual departure point—their expectation that unchanging covariance measures across blocks implies unchanging ΔC and ΔN—is not correct. As emphasized above, variance and covariance are computed from execution variables. The components ΔC, ΔN, and ΔT are calculated in result space, in their respective dimensions (meters in the darts example).

Different Coordinate Frames Yield Different Results

Despite this problem, Smeets and Louw are correct in pointing out that the TNC analysis yields different results in different coordinate systems. In fact, as we proceed to show, it is not only the choice of the coordinate system of the execution variables but also the choice of the result variable(s) that determines the relative contribution of the three components ΔT, ΔN, and ΔC. Even further, this choice of variables affects all three components, not solely covariation, as Smeets and Louw point out. We elucidate this issue with another simulation of a simple task in which a two-dimensional target area is hit in a sequence of trials.

The result in this task is described by two-dimensional Cartesian coordinates (x, y), where the target area is a parallelogram. The origin is at the center of the parallelogram and the coordinates of the corners are $(-1, 0)$, $(0, 2)$, $(1, 0)$, and $(0, -2)$. Any result within the target area is assigned the result measure $R_i = 1$; any result outside the target area is assigned $R_i = 0$, according to the definition

$$R_i = f(x_i, y_i) = \begin{cases} R_i = 1 \Leftrightarrow & |y_i| + |2x_i| < 2 \\ R_i = 0 \Leftrightarrow & |y_i| + |2x_i| \geq 2 \end{cases}. \quad (3)$$

In order to simulate a learning process, we generated two blocks of data (B_1 and B_2) with 100 trials each. The execution variables (x_i, y_i) of the two blocks have different means and standard deviations. The result measure R for each block, defined as the percentage of success within 100 trials, shows an increase from B_1 to B_2 (Figure 1). To demonstrate the dependency of the calculations of ΔT, ΔN, and ΔC on different coordinate systems, we transformed the data into two alternative coordinate descriptions from the original execution variables. The first transformation was a rotation of the coordinates by $\pi/4$ rad; the second was a transformation into polar coordinates:

1. Rotation by $\pi/4$: $(x_i', y_i') = f_R(x_i, y_i)$, where

$$x_i' = x_i \cos(-\pi/4) - y_i \sin(-\pi/4) \text{ and}$$
$$y_i' = x_i \sin(-\pi/4) + y_i \cos(-\pi/4). \quad (4)$$

2. Polar coordinate system with its origin at (0, 2.1) of the original coordinate system:

$$(r_i, \phi_i) = f_P(x_i, y_i), \text{ where } r_i = (x_i^2 + (y_i - 2.1)^2)^{0.5} \text{ and}$$
$$\phi_i = \arctan(x_i/(y_i - 2.1)). \quad (5)$$

The same transformations were applied to B_1 and B_2. Figure 1 illustrates the sequence of steps in the TNC calculations for the three coordinate systems. The gray areas depict the target area in the respective coordinates of the execution variables together with their coordinate axes. Hence, the target takes on different shapes in the three coordinate systems following Equation 3. The first and fifth data sets in each row show the distributions of the 100 trials of B_1 and B_2 in their respective execution variables. Note that the results R of B_1 and B_2 are identical in all three coordinate descriptions: 30 and 88, respectively. The means of the distributions are depicted by the bold dots. The three intermediate data sets in each row represent the computational steps in extracting the three components (for details see Müller & Sternad, 2004b): B_1-P and B_2-P represent the permutations of B_1 and B_2. B_1-P-shift contains the same distribution as B_1-P but shifted to the mean location of B_2. The bar charts on the right display the results when expressed by two different result measures R_1 and R_5, described below.

Contrasting the calculation steps in the three coordinates shows that permutation and mean shifting affect the result differently in the different coordinate frames, because the means of B_1 and B_2 have different locations with respect to the coordinates. First, the effect of the permutation is much larger in the rotated coordinate system compared to the original and the polar coordinate systems and, hence, produces a larger contribution of covariation ΔC. Second, the shift in the mean from B_1 to B_2 in the polar system effectively leads to a reduced dispersion in the angle variable. This is due to the fact that a given Cartesian distance becomes an increasingly smaller angular distance the farther it is away from the polar origin. Hence, the contribution of noise ΔN is signifi-

Table 2
Results of the Tolerance-Noise Covariation Method When Applied to Dart Throwing, Described in Cartesian Coordinates

Compared blocks	ΔD (m)	ΔC (m)	Δcov	ΔT (m)	ΔN (m)	ΔSD_x	ΔSD_y
$B_1 \rightarrow B_2$	−0.370	0.049	0.000	−0.321	−0.098	0.000	0.000
$B_2 \rightarrow B_3$	−0.107	0.009	0.000	−0.099	−0.017	0.000	0.000

Note. The performance improvement is quantified by the difference in the result variable D across blocks. ΔC, ΔT, and ΔN denote the contributions of covariation, tolerance, and noise, respectively. Changes in the standard deviations (ΔSD_x and ΔSD_y) and changes in covariance (Δcov) are not identical to the components ΔN and ΔC. ΔC refers to the difference in covariation across blocks, for example, $\Delta C(B_2) - \Delta C(B_1)$.

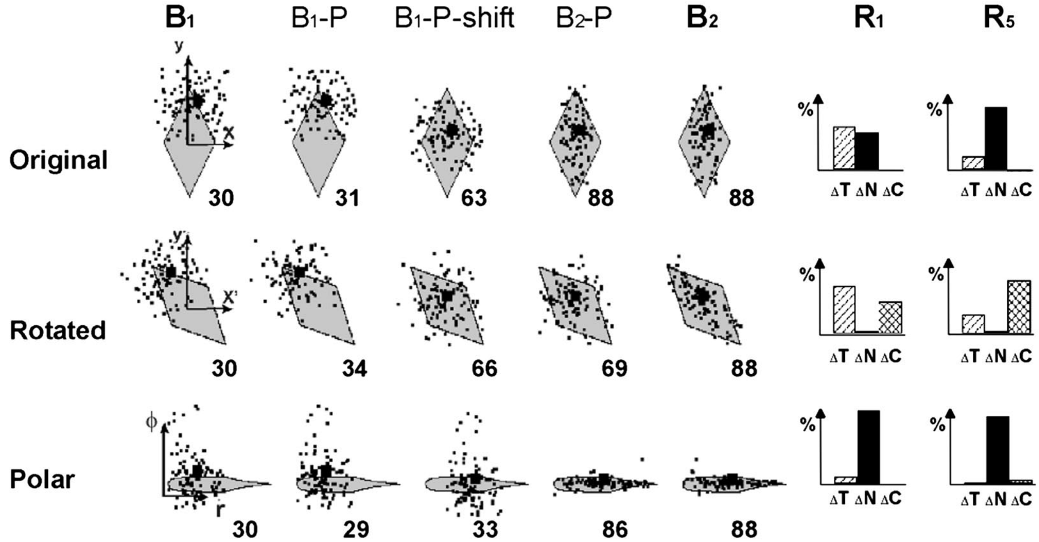

Figure 1. Effects of coordinates on the TNC (tolerance-noise-covariation) decomposition. The top row shows the original data, defined by Equation 3. The second row shows the same data in a coordinate system rotated by π/4 rad. The third row depicts the same data in a polar coordinate system with the origin shifted to (0, 2.1). The gray areas represent the target area or result space, and the black dots represent the individual trials. Data points inside the gray area are successful trials, assigned the value 1; data points outside this area are assigned the value zero. The successive data sets in each row show the permuted data from Block 1 (B_1-P), the data after shifting by the mean (B_1-P-shift), and, lastly, the permutation of the data in Block 2 (B_2-P). The two bar charts to the right of each row express the results of the TNC decomposition for the two different result measures, R_1 and R_5.

cantly increased in the polar compared to the original coordinate system. In sum, the differences in the TNC analysis results in the three different coordinate systems are the consequence of the permutation and shifting at the level of the execution variables, and performance is always evaluated at the level of the results.

The TNC calculations were also conducted for two different performance measures. R_1 describes the average hitting percentage in 100 trials: $R_1 = 30$ in B_1, and $R_1 = 88$ in B_2. Alternatively, for the same data sets a result measure R_5 was defined as the percentage in 100 trials that five successful trials occur in sequence. In our example this corresponds to $R_5 = .243$ for the same B_1 and $R_5 = 53$ for B_2. As the bar charts in Figure 1 illustrate and Table 3 summarizes, this choice of result measures has a significant effect on the outcome of the TNC analysis.

It is important to keep in mind that this dependency on the choice of coordinate system is a feature of many other quantities. As Smeets and Louw state themselves, elementary statistical measures such as means and standard deviations are also not invariant under nonlinear coordinate transformations. Evidently, this fact affects many areas of research in motor control and learning. Although rarely reflected upon, the choice of variables determines the pattern of results. Smeets (2000) himself has pointed out that the time course of a learning curve is dependent on the chosen measure, for example, movement time or average velocity. This is critical to keep in mind because the time course of the learning curve, whether it follows a power law or an exponential time course, has been the focus of substantial discussion (Heathcote & Brown, 2004; Liu, Mayer-Kress, & Newell, 2003; Liu, Mayer-Kress, & Newell, 2004; Schmidt & Lee, 1999).

This issue can be reinforced with the same simulation example. If one assumes that the data sets B_1 and B_2 are part of a sequence of practice trials, B_1 representing an early and B_2 a late stage with improved performance, the continuous learning process can be plotted over time. Assuming an exponential progression of change

Table 3
Contributions of the Components ΔC, ΔT, and ΔN to Performance Improvement (ΔR) in the Three Different Coordinate Systems for Two Alternative Result Measures (R_1 and R_5)

Coordinate system	R_1				R_5			
	ΔR	ΔC	ΔT	ΔN	ΔR	ΔC	ΔT	ΔN
Original	0.58	0.32	0.25	0.01	0.53	0.10	0.43	0.00
Rotated	0.58	0.32	0.03	0.23	0.53	0.12	0.03	0.37
Polar	0.58	0.04	0.53	0.01	0.53	0.00	0.47	0.06

Note. ΔC, ΔT, and ΔN denote the contributions of covariation, tolerance, and noise, respectively.

from B_1 to B_2 for the result measure R_1, Figure 2 schematically illustrates the learning curve; large improvements in the beginning approach an asymptote later. When this rate of change is transformed into the result measure R_5 for exactly the same data sets B_1 and B_2, the time course of improvement for the same data indicates that the rate of learning is first accelerating and later decelerating.

Context Dependency of TNC Components is a Useful Feature to Evaluate Different Coordinate Systems

Whereas Smeets and Louw interpret this feature as a problem specific to the TNC method, we maintain that it is (a) not specific to the TNC method and (b) a valuable feature of a method intended to examine issues of control.

A glance into basic mechanics emphasizes the importance of the choice of coordinate system even in physics. Consider an example of a child sitting on a merry-go-round that rotates at constant speed. From the perspective of an observer on the ground she is continuously accelerating and decelerating, even as she sits still at a constant radius from the axis of rotation—each coordinate of a Cartesian reference frame exhibits a sinusoidal variation with time, and the child is said to be subject to an inertial force proportional to acceleration. Alternatively, her motion may be described in polar coordinates by two variables, one of which (radius) is constant, whereas the other (angle) varies linearly with time. In this coordinate frame it appears that she does not accelerate, as the second time derivative of both variables is zero. Nonetheless, she experiences a centrifugal force (proportional to the square of angular velocity) accelerating her in the radially outward direction. To describe the same phenomenon in polar coordinates, we note that the equations of motion include a centrifugal force but no inertial force, whereas in the nonrotating Cartesian coordinate system they include an inertial force but no centrifugal force. The important point is that both descriptions are correct and can be transformed into each other. Analogously, in the darts example of Smeets and Louw the Cartesian coordinate system describes the change due to learning in terms of a change in ΔT, whereas the same change described in polar coordinates renders ΔN and ΔT as primary. Both descriptions are correct.

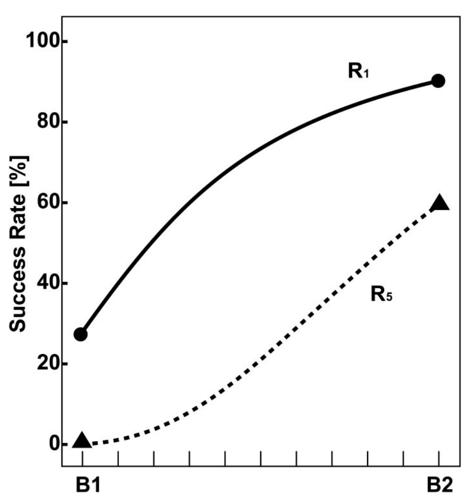

Figure 2. Two schematic learning curves derived from the same process but described in two different result measures, R_1 and R_5. B_1 = Block 1 data set; B_2 = Block 2 data set (each consisting of 100 trials).

Many if not all fundamental movement phenomena similarly depend on the coordinate systems in which they are described: The universally accepted and robust observations of Fitts law or the 2/3 power law suffer from the same feature, even though alternative coordinate descriptions may not be as obvious. Do we similarly have to call into question whether these observations are flawed, as they do not show universality across coordinate descriptions? Curiously, this point has been little reflected upon in movement science, and therefore, Smeets and Louw raise an important issue. However, the next logical questions are what constitutes the "correct" choice of coordinate system and whether there is a "proper" coordinate system for any given phenomenon.

With Smeets and Louw's example of how the statistical mean changes and loses its "correct" meaning after a nonlinear (logarithmic) transform of the originally normally distributed data, they implicitly suggest that a normal distribution of variation might serve as the criterion for the correct choice of variable. Although this seems practical and reasonable, it cannot be considered a universal criterion to identify the coordinates of neural control processes. From the central limit theorem we know that a normal distribution of variability results from processes that sum a sufficient number of independent random effects. Though this is a good model of many noise processes, it is by no means universal; for example, uniform distributions are often more appropriate.

In the present context the issue is even more complex. Because nonlinear relations between execution and result variables are common, some variables may obey a normal distribution, whereas others may not. Should execution variables be normally distributed or result variables? Even in the simple example of dart throwing, given a nonlinear deterministic relation between execution variables and results, only one of these set(s) of variables can satisfy this criterion. In practice, it is more likely that none of them will. Furthermore, measurement noise may also have a normal distribution, and this may mask the effects of underlying neural control processes. Therefore, the distribution properties of a variable cannot provide the only criterion for its validity as a focus of neural control.

These considerations highlight the importance of coordinate system choice in all areas of motor control. In their comparison of Cartesian and polar coordinates, Smeets and Louw imply that the "more complex" results rendered by the polar description make it less appropriate for describing the change in performance. A different rationale is exemplified by studies testing the hypothesis of the uncontrolled manifold (UCM) in redundant tasks (Scholz & Schöner, 1999; Scholz, Schöner, & Latash, 2000). Analyzing variability in execution space only, the UCM hypothesis makes use of the fact that specific values of the result variable define different "uncontrolled manifolds" in execution space—combinations of execution variables that do not affect the result. Based on the hypothesis that different UCMs represent different variables controlled by the nervous system, the criterion for favoring one or the other "controlled variable" is that execution variance is structured predominantly parallel to the manifold, that is in the "don't care" direction. Although this appears to provide an intuitively appealing and rigorous example of hypothesis testing, it is important to realize that the results of UCM analysis also depend heavily on the choice of execution or elemental variables, just as the TNC method does. In particular, in their application to multijoint limb movements in 3-D, several reasonable, practical definitions of joint angles are available, some of which (e.g., whether joint angles are defined relative to adjacent limb segments or to a common reference frame such as the thorax) are related by linear transformations. Consequently, the results of UCM analysis depend sensitively on the experimenters' choice of joint angles, including whether execution variance exhibits any structure at all.

One of the most fundamental questions in motor control concerns coordinate systems: Is the coordinate system for biological control centered in the body, in extrinsic space, or is it oculocentric (Buneo, Soechting, & Flanders, 2002; Engel, Flanders, & Soechting, 2002) or some combination of all of these possibilities? Is the space Euclidean or affine (Pollick, 1997)? Does control of limb movements follow Listing's law (Lieberman, Biess, Friedman, Gielen, & Flash, 2006)? The structure and shape of variability may provide clues to which coordinate system is relevant for human actors. It is reasonable to expect that the coordinate system in which variability is systematic is the most likely candidate. As such, TNC analyses in different coordinates may become a tool to evaluate different candidate systems for their relevance to the human actor.

Summary and Conclusions

The calculation of covariation is not invariant with respect to coordinate transformations as pointed out by Smeets and Louw. For nonlinear coordinate transformations, the other two components ΔT and ΔN are also affected. However, this lack of invariance is similarly inherent to many other measurement and analysis methods. Despite this apparent shortcoming, the question of the relevant variables is one of the most fundamental in movement science. What is needed are strategies and methods to find and evaluate the coordinate system(s) in which the biological system generates and optimizes its control and coordination. With this problem in mind, comparison of results as performed in different coordinate systems may provide one way to see where invariances in performance lie.

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